Random and Periodic sleep schedules for target detection in sensor networks

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***Abstract*—We study Random and Periodic sleep schedules from the point of view of delay in detecting the target. We consider sleep schedules in which a sensor in 'inactive' mode wakes up either randomly or periodically to detect presence of the target within its vicinity resulting into two sleep schedules viz. (a) Random wake-up schedule, and (b) Periodic wake-up schedule respectively. Specifically, we analyse and obtain for the Random wake-up schedule the expected delay in detection, and the delay, such that with probability P, the delay is less than the computed value. For the Periodic wake-up schedule we show that there exists an upper bound on the delay. Further we compute the average value of delay. We have shown that the theoretically computed averages and the upper bounds on the delay match with the simulation results for the Random wake-up and Periodic wake-up schedules.**

I. Introduction

In this paper, we consider the problem of a given sensor detecting the presence of a target soon after it moves into the vicinity of the sensor. Once that happens, the sensor may, for instance, make measurements to/from target or take any other suitable action. This is particularly relevant in an application where a network of sensors have the responsibility of tracking a moving target in a 2-D plane. This is done as follows: as the target moves, an adequate number of sensors within its vicinity detect the presence of the target and make suitable distance measurements. Then, together they compute the location of the target using trilateration, for instance.

A sensor is a tiny device consisting of a CPU, a capability to communicate data packets, and to sense its environment [1], [2], [3]. Typically, the sensors are powered from a battery source and, unless its energy is conserved, the life span of the sensor can be as little as a few days. Therefore, a sensor must be programmed to follow a schedule where it enters the 'inactive' mode, during which it is asleep most of the time. It periodically or randomly 'wakes up' to determine whether it must exit the 'inactive' mode and enter an 'active' mode. In an 'active' mode, a sensor is fully functional and is capable of sensing the environment, transmitting and receiving data, measuring distance to a target, etc., and to determine if and when to exit the 'active' mode and re-enter 'inactive' mode. Table I illustrates consumption of energy in active or inactive modes of operation of a typical sensor.

In this paper, given that a sensor is focused on detecting the presence of a target, we assume that the sensor follows a pre-determined schedule where it shuts down its transceiver for most of the time during its 'inactive' mode, but wakes up periodically, or at random intervals (see Figure 1), by turning on its transceivers to possibly detect radio transmissions from a target if and when it is within the vicinity of the target. Once the target is detected the sensor immediately exits the 'inactive' mode to enter the 'active' mode where it remains 'awake', ready to perform other actions (see Figure 1).

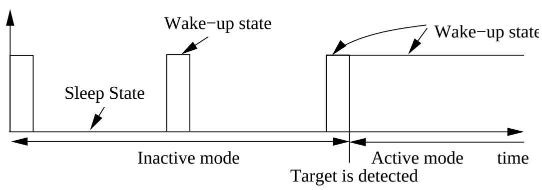

Fig. 1. Active and inactive modes.

A sensor while it is awake in 'inactive' mode detects the presence of the target in its vicinity when it receives a 'beacon' that consists of N bits comprised of a preamble (of n_1 bits) and a target identifier (of n_2 bits)(see Figure 2). Necessarily, the duration, δ_s, for which the sensor must be awake is such that $\delta_s > \delta_t$, where $\delta_t = \frac{N}{C}$ is the time taken by the target to transmit the beacon, $N = n_1 + n_2$, and C is the transmission rate of the target. If

Mode / operation	Consumption
Active	
- Idling	8 mA
- Transmission	27 mA
- Receiving	10 mA
Inactive	
- Wake-up	8 mA
- Sleep	less than 15 μA

TABLE I
CURRENT CONSUMPTION BY SENSOR NODE IN DIFFERENT STATES [3].

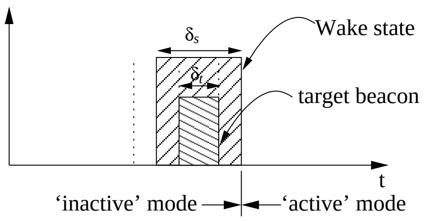

Fig. 2. Overlap of 'wake' state and target beacon.

the sensor wakes up at any time during the period of length upto $\varepsilon = \delta_s - \delta_t$ just before the target starts to send a beacon, then it captures the beacon completely as shown in Figure 2.

We assume that the target transmits beacons periodically after every T_t time, as in Figure 3 (a), while a sensor wakes up either periodically, as in Figure 3 (b), or randomly, as in Figure 3 (c). In case a sensor wakes up *periodically* as in Figure 3 (b), the time period after which it wakes up is T_s. In case a sensor wakes up *randomly*, as in Figure 3 (c), T_s is the mean of time τ_s after which it wakes up. That is, $T_s = E(\tau_s)$. Typically $\delta_t < T_t$ while $\delta_s << T_s$.

We now define the delay in detecting the presence of the target by a sensor, Δ as

$$\Delta = t_2 - t_1, \tag{1}$$

where t_1 is the time when the target first transmits a beacon after it appears within the range of the sensor and t_2 is the time when the sensor wakes up to receive and process a beacon from the target (see Figure 3 (b) and 3 (c)).

In the following sections, we develop and describe two schedules, viz. (a) Random wake-up schedule, and (b) Periodic wake-up schedule. We also analyse and obtain for the Random wake-up schedule the expected delay in detection, viz. $E(\Delta)$, and the delay, $E(\Delta^P)$, such that with probability P, $\Delta \leq E(\Delta^P)$. The latter gives an assessment of how large the delay can potentially be.

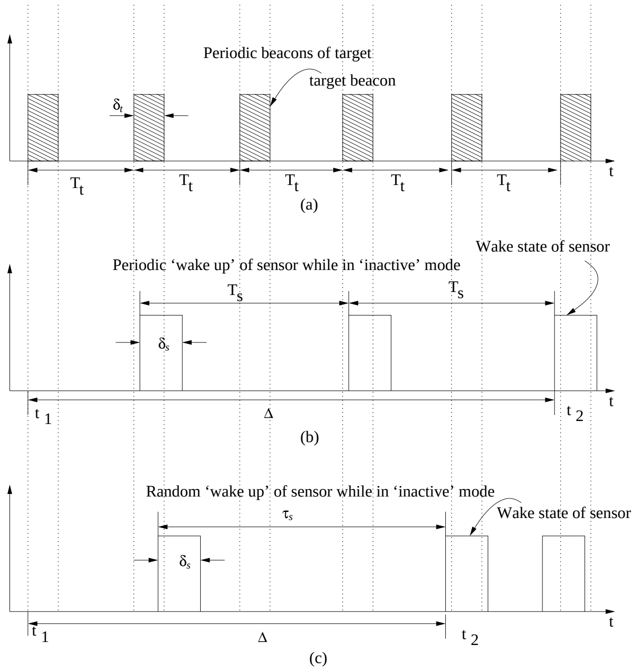

Fig. 3. Periodic beacons of the target and random or periodic 'wake-up' state of sensor.

For the Periodic wake-up schedule we first establish that there exists an upper bound, Δ_{max}, on the delay, provided that the target and the sensor are synchronised. We relax the requirement of synchronisation later. We subsequently compare the two schemes.

II. RELATED WORK

Several researchers have in the past considered minimizing consumption of energy by programming the sensor nodes to follow a specific sleep schedule consisting of an inactive (or sleep) mode followed by an active mode. The contexts in which such works have been reported have varied. In one set of papers, the major concern has to do with putting a sensor to sleep when it is not expected to either sense the environment, or to transmit or receive data. In such a situation, data transfer will succeed only when both the transmitter and the corresponding receiver are simultaneously in active mode. Therefore, in paper by Ye et al [4], for instance, the entire effort is to synchronize the sleep schedules of corresponding sensors (this could be pairs of sensors or a cluster thereof).

In the paper by Liu et al [5], energy consumption is proposed to be minimized by suitably coordinating data transfers so that collisions are minimized. This is achieved by assigning different colors, or sleep intervals, to sensors along a data path. Further, sensors that are not on the data path work with significantly reduced duty cycles. The work reported in a paper by Li et al [6] also proposes a protocol to schedule data transfers so as to avoid excessive collisions while trying to minimize the time to collate data from all sensors in the network. The paper by Hohlt et al [7] also describes a method to adapt the sleep schedule of sensors in accordance with the demand for data transfer. As before, this paper is also not concerned with event detection (or target tracking).

The papers by Tian et al [8], Xu et al [9], Ye et al [10], Gui et al [11], and Hsin et al [12] are concerned with detection of an event soon after it occurs within a given region, the latter being defined by the sensing range specific to the event and its detection. In order to ensure that the event is detected with no delay whatsoever, most schemes suggest that there be a redundant set of sensors, at least one of which is awake at the time of occurrence of the event. Therefore, in most of these papers, the effort is geared towards developing a schedule for dynamically putting to sleep all but one sensor (in some cases many sensors). In doing so these papers evaluate both random and coordinated schemes, as well as the trade-off between the extent of redundancy vs. the probability of ensuring that at least one sensor in every region is awake. Clearly, the resulting delay in detecting an event is zero. In the case where the events occur rarely, Cao et al [13] propose that even during the time when a sensor is awake, it deactivates its sensing mechanism for some time. This deactivation/activation is possibly coordinated so as to minimize the average detection delay. Note that in all the above papers it is a fundamental assumption that if an event occurs in a given region then it can be detected instantaneously provided a sensor in the region is awake and its sensory mechanisms are activated.

Our paper is concerned with tracking a moving target. The immediate problem, however, is to detect its presence by a sensor soon after it appears within the region defined by the detection (or radio) range as determined by the strength of the beacon and the sensitivity of the detector in the sensor. Inherent in this assumption is that the target cooperates with the network of sensors by sending a beacon either continuously or after periodic intervals. The case where the beacon is sent continuously is trivially simple since the resulting average delay is the average duration for which the sensor is put to sleep. In case the beacon is sent periodically (or using a different schedule) to avoid blocking a channel then the resulting delay is significantly, and in a complex manner, dependent upon the sleep schedule of the sensor. Clearly, the target is said to be detected if the beacon is received

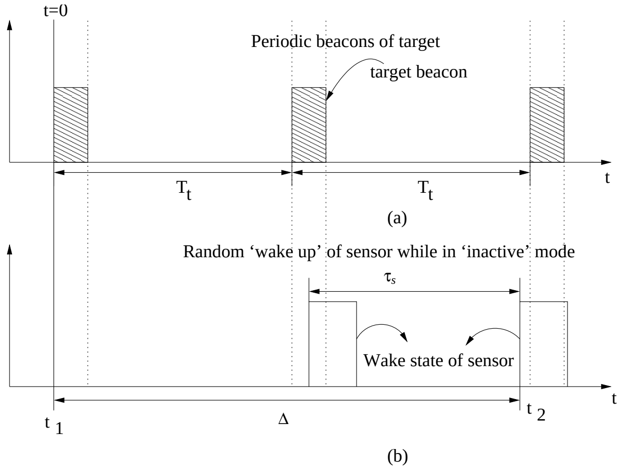

Fig. 4. Periodic beacons of the target and random 'wake-up' state of sensor.

during the time the sensor is awake but not otherwise.

While most papers refer to multiple sensors per region, and to one or more regions, we make no such assumption. In the context of the application considered in our paper a number of sensors are deployed (in some random or systematic fashion) in a 2-D space, and each sensor is expected to detect the presence of the target soon after it appears in the vicinity of the sensor. There is, as a result, no need for coordination of sleep schedules of various sensors. No doubt, the delay vs. duty cycle characteristics can be improved if one were to dynamically change the sleep schedule to one with a larger duty cycle if the target was known to have been detected by any of its neighboring sensors. (The latter is, however, beyond the scope of this paper.)

III. Random wake-up schedule for detection of a target

In this schedule, a sensor wakes up randomly. The mean time after which it wakes up is assumed to be *exponentially distributed* with a mean $T_s = \frac{1}{\mu}$ (A similar analysis can be done for other probability distributions as well).

We now obtain an analytical expression for delay, Δ, in detecting the presence of the target by a sensor. Before doing so, we define $t = 0$ as the time when the target first transmits a beacon soon after it appears within the radio range of the sensor.

A. Probability of success on first attempt

Let p_1 be the probability of success in detecting a target in its first attempt. A " first attempt" is charac-

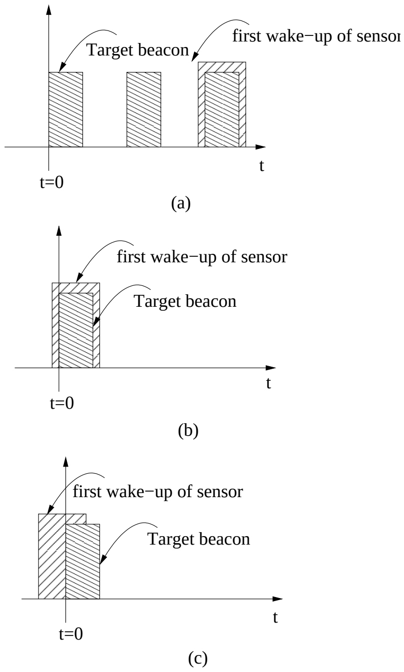

Fig. 5. First attempt by the sensor to detect the target beacon.

terised by the first wake-up interval ending at time $t = 0$ *or later*. As an example, see Figures 5 (a), 5 (b), and 5 (c). Clearly, p_1 is dependent on (a) the periodicity of target beacons, T_t, (b) the mean time, T_s, after which the sensor wakes up to attempt to detect the target, and (c) the time duration, δ_s for which the sensor remains awake while the beacon is of duration δ_t.

Consider obtaining a value for p_1. At $t = 0$, the sensor may be (a) awake, or (b) asleep. The probability that the sensor is awake is, clearly, $\frac{\delta_s}{T_s}$, and that of sensor being asleep is $(1 - \frac{\delta_s}{T_s})$. The probability, p_1, is therefore

$$p_1 = \frac{\delta_s}{T_s} q_1 + \left\{1 - \frac{\delta_s}{T_s}\right\} q_2, \tag{2}$$

where q_1 is the probability that the sensor detects the first beacon, given that it is awake at $t = 0$ (see Figures 5 (b) and 5 (c)), and q_2 is the probability that the sensor detects a beacon sent at time $t > 0$, given that the sensor is asleep at $t = 0$, as in Figure 5 (a).

If the sensor is awake at $t = 0$, it will detect and interpret correctly the first beacon if it wakes up sometime during the time interval $[-\varepsilon, 0)$ where $\varepsilon = \delta_s - \delta_t$ (as in Figure 5 (b)). The probability of doing so is $\frac{\varepsilon}{\delta_s}$. If it wakes up within the time interval $(-\delta_s, -\varepsilon)$ then, surely, the sensor will not detect the first beacon (as in Figure 5 (c)). As a consequence,

$$q_1 = \frac{\varepsilon}{\delta_s}. \tag{3}$$

Clearly, if the sensor is asleep at $t = 0$, then the first beacon at $t = 0$ will *not* be detected. The sensor may detect a beacon when it wakes up next (at $t > 0$). The probability of doing so, q_2, is

$$q_2 = \sum_{j=1}^{j=\infty} \int_{jT_t-\varepsilon}^{jT_t} \mu e^{-\mu y} dy, \tag{4}$$

$$= \frac{(e^{\mu\varepsilon} - 1)}{(e^{\mu T_t} - 1).} \tag{5}$$

Thus,

$$p_1 = \frac{\varepsilon}{T_s} + \left\{1 - \frac{\delta_s}{T_s}\right\} \left\{ \frac{(e^{\mu\varepsilon} - 1)}{(e^{\mu T_t} - 1)} \right\}. \tag{6}$$

Recall that we have assumed that if the sensor is asleep at $t = 0$ then the time when it wakes up next is exponentially distributed with a mean T_s. Further, $\mu = \frac{1}{T_s}$. This also assumes that the wake up schedule is memory-less, or that the time it wakes up next does not depend upon when it was awake the last time.

Example 1: Consider the case when $T_t = 10$, $T_s = 100$ (or that $\mu = \frac{1}{100}$), $\delta_t = 1$, $\delta_s = 2$. Then, $\varepsilon = \delta_s - \delta_t = 1$, $\frac{\delta_s}{T_s} = 0.02$, $\frac{\varepsilon}{\delta_s} = 0.5$. Thus, $q_1 = 0.5 \times 1$, and $q_2 = 0.0956$. As a result, $p_1 = 0.1037$.

B. Probability of success on subsequent attempts

We now consider computing the probability, p, of detecting the target on its second, third, or subsequent attempts, independent of success/failure of any earlier attempt(s). The k^{th} attempt (see also Figure 6 (a)) is characterised by the sensor waking up at a time $t > t^{k-1}$, where t^{k-1} is the time when the sensor wakes up on its $(k-1)^{th}$ attempt. The figure also brings out the fact that t^{k-1} is b away from the time when the last beacon was transmitted. For a given b, $0 \le b \le T_t - \varepsilon$, the probability of detecting the beacon is

$$\sum_{j=1}^{j=\infty} \int_{jT_t-\varepsilon-b}^{jT_t-b} \mu e^{-\mu y} dy, \tag{7}$$

else when $T_t - \varepsilon \le b \le T_t$, the probability of detecting the beacon is

$$\int_0^{T_t-b} \mu e^{-\mu y} dy + \sum_{j=2}^{j=\infty} \int_{jT_t-\varepsilon-b}^{jT_t-b} \mu e^{-\mu y} dy. \tag{8}$$

Now, since $0 \le b \le T_t$,

$$p = \int_{b=0}^{T_t} \sum_{j=1}^{j=\infty} \int_{\max\,(jT_t-\varepsilon-b,0)}^{jT_t-b} (\mu e^{-\mu y} dy) \frac{db}{T_t}. \tag{9}$$

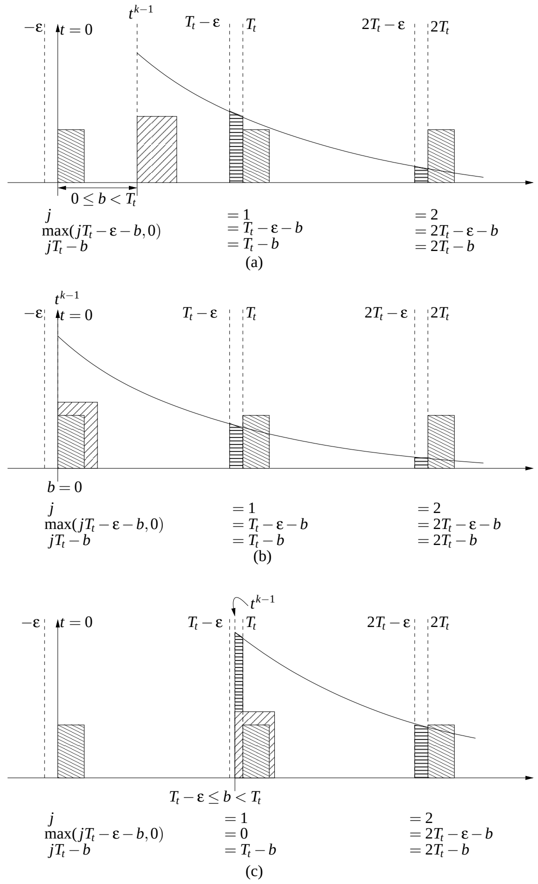

Fig. 6. p, the probability that the sensor detects a target in its next attempt.

The above is valid since b is random and uniformly distributed over $[0, T_t]$. The latter assumption is justified because in the absence of any other information, the sensor would have woken up at any time between two successive beacons.

Thus the probability, p, of detecting a target in its next attempt is given by:

$$\begin{aligned} p &= \int_{b=0}^{T_t} \int_{\max\,(T_t-\varepsilon-b,0)}^{T_t-b} (\mu e^{-\mu y} dy) \frac{db}{T_t} \\ &+ \int_{b=0}^{T_t} \sum_{j=2}^{j=\infty} \int_{jT_t-\varepsilon-b}^{jT_t-b} (\mu e^{-\mu y} dy) \frac{db}{T_t} \\ &= \int_{b=0}^{T_t-\varepsilon} \int_{T_t-\varepsilon-b}^{T_t-b} (\mu e^{-\mu y} dy) \frac{db}{T_t} \\ &+ \int_{b=T_t-\varepsilon}^{T_t} \int_{0}^{T_t-b} (\mu e^{-\mu y} dy) \frac{db}{T_t} \\ &+ \int_{b=0}^{T_t} \sum_{j=2}^{j=\infty} \int_{jT_t-\varepsilon-b}^{jT_t-b} (\mu e^{-\mu y} dy) \frac{db}{T_t} \\ &= \frac{\varepsilon}{T_t} \end{aligned} \tag{10}$$

In other words, independent of the outcome of the earlier attempts, the probability of success[1] on any attempt is $\frac{\varepsilon}{T_t}$.

Given that p_1 is the the probability of success in detecting the target on the first attempt, and p is the probability of success on any subsequent attempt, the probability that the target is detected on the second attempt, $p_2 = (1-p_1)p$, and on the third attempt, $p_3 = (1-p_1)(1-p)p$, and so on. Thus, the probability that exactly i attempts are required to detect the target, $p_i(i \geq 2) = (1-p_1)(1-p)^{(i-2)}p$.

C. Results

Theorem 3.1: Given p_1, and p, the expected number of attempts required by the sensor to detect the target, $\mathrm{E}(k)$, is

$$E(k) = p_1 + (1-p_1)\left[\frac{p^2-1}{(p-1)p}\right]. \tag{13}$$

Further, the expected delay for a sensor to detect the target, $E(\Delta)$, is

$$E(\Delta) = \left[p_1 + (1-p_1)\left\{\frac{p^2-1}{(p-1)p}\right\}\right] T_s. \tag{14}$$

Proof: Given p_1 and p, the probability that exactly i attempts are required to detect the target, $p_i(i \geq 2) = (1-p_1)(1-p)^{(i-2)}p$. The expected number of attempts required to detect the target is

$$E(k) = \sum_{i=1}^{\infty} i p_i. \tag{15}$$

As a result, $\mathrm{E}(k)$ is given by Equation 13. Further, if τ_i is the duration between two successive attempts and k is the number of attempts required to detect the target,

$$E(\Delta) = E\left(\sum_{i=1}^{i=k} \tau_i\right). \tag{16}$$

Using Wald's equation [14],

$$E(\Delta) = E(\tau_i) E(k). \tag{17}$$

As a result, $\mathrm{E}(\Delta)$ is given by Equation 14. ∎

[1] A similar analysis done for uniform probability distribution, shows that

$$\begin{aligned} p_1 &= \frac{\varepsilon}{T_s} + \left\{1 - \frac{\delta_s}{T_s}\right\}\left\{\frac{\varepsilon}{T_t}\right\}, \qquad (11) \\ p &= \frac{\varepsilon}{T_t}. \qquad (12) \end{aligned}$$

Here, we assume that T_s is a multiple of T_t, that is, $T_s = mT_t$.

While there does *not* exist an upper bound on the delay in detecting the target, it is possible to compute $\mathrm{E}(\Delta^P)$, such that with probability, P, the delay is at most $\mathrm{E}(\Delta^P)$.

Theorem 3.2: Given p_1, and p, the sensor will detect the target with probability P in k or fewer attempts, where

$$k = \frac{\log\left[\frac{1-P}{1-p_1}\right]}{\log\,(1-p)} + 1. \tag{18}$$

Further, with probability P, the delay is at most

$$E(\Delta^P) = \left[\frac{\log\left[\frac{1-P}{1-p_1}\right]}{\log\,(1-p)} + 1\right] T_s. \tag{19}$$

Proof: Given p_1, and p, the probability, P, that k or fewer attempts are required is

$$P = \sum_{i=1}^{i=k} p_i \tag{20}$$

$$= p_1 + \sum_{i=2}^{k} (1-p_1)(1-p)^{i-2} p \tag{21}$$

$$= 1 - (1-p_1)(1-p)^{k-1} \tag{22}$$

As a result, the probability that k or fewer attempts are required, is given by Equation 18. Further, with probability P, the expected delay encountered is at most $\mathrm{E}(\Delta^P)$ given by Equation 19. ■

We now summarize the Random wake-up schedule for target detection.

Random wake-up schedule: Let the target send beacons of duration $\delta_t = \frac{N}{C}$, and consisting of N bits at C bps, and with periodicity, T_t. The sensor wakes up for a duration δ_s, at random time intervals such that time between two wake-up states is exponentially distributed with mean value of T_s. Then

- the expected number of attempts, $\mathrm{E}(k) = p_1 + (1-p_1)\left[\frac{p^2-1}{(p-1)p}\right]$,
- the expected delay in detection is $\mathrm{E}(\Delta) = \left[p_1 + (1-p_1)\left\{\frac{p^2-1}{(p-1)p}\right\}\right] T_s$,
- with probability P, the target will be detected before a delay of at most $\mathrm{E}(\Delta^P) = \left[\frac{\log\left[\frac{1-P}{1-p_1}\right]}{\log\,(1-p)} + 1\right] T_s$, and
- the resulting duty cycle of the sensor, or the energy consumed by the sensor per unit time, $\eta_s = 2\left(\frac{N}{C}\right)\left(\frac{1}{T_s}\right)$.

D. Discussion and analysis

Example 2: Consider the earlier example when $T_t = 10$, $T_s = 100$, $\delta_t = 1$, $\delta_s = 2$. Then, $\varepsilon = \delta_s - \delta_t = 1$, and $p_1 = 0.1037$, $p = 0.1$, $p_2 = 0.0906$, $p_3 = 0.08$, etc. Further,

- the expected number of attempts to detect the target, $E(k) = 9.96$, and the corresponding expected delay to detect the target, $E(\Delta) = 996$,
- with probability $P = 0.95$, the number of attempts required to detect the target is at most $k = 28.39$, and the corresponding delay is at most $E(\Delta^P) = 2839$, and
- the duty cycle, $\eta_s = \frac{\delta_s}{T_s} = 0.02$.

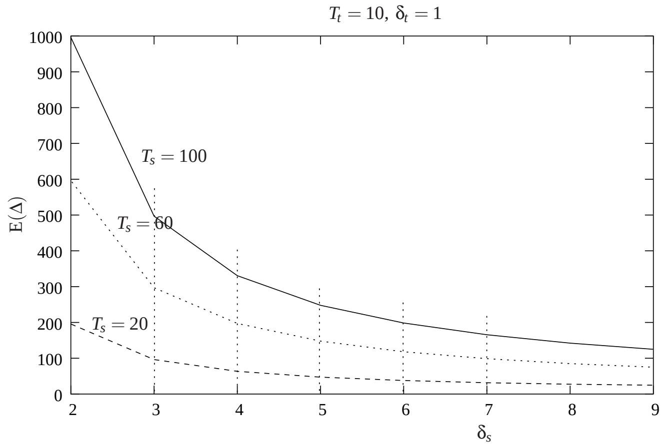

Fig. 7. $\mathrm{E}(\Delta)$ vs. δ_s.

The computed value of $\mathrm{E}(\Delta)$ vs. δ_s (for different T_s) is given in Figure 7. From Figure 7, we note that for a fixed T_s, $\mathrm{E}(\Delta)$ reduces as δ_s increases and for a fixed δ_s, $\mathrm{E}(\Delta)$ increases (linearly) with T_s. This is to be expected since as T_s increases the attempts are made less frequently. Further, as δ_s increases, the probability of success on any given attempt increases.

Further, for probability $P = 0.95$, the computed value of $\mathrm{E}(\Delta^P)$ vs. δ_s (for different T_s) is given in Figure 8. From Figure 8, we note that for a fixed T_s, $\mathrm{E}(\Delta^P)$ reduces as δ_s increases, and for a fixed δ_s, $\mathrm{E}(\Delta^P)$ also increases (linearly) with T_s.

Taking this analysis one step further, we plot $E(\Delta)$ vs. $\eta_s = \frac{\delta_s}{T_s}$ for different values of T_s in Figure 9 to illustrate the trade-off between the duration for which the sensor is awake, δ_s, and the periodicity with which the sensor wakes up, T_s. In Figure 9, for example, points A, B, C, and D correspond to different values of duty cycle that result in the same expected delay $\mathrm{E}(\Delta) = 200$. A, B, C, and D have duty cycle of 0.06, 0.067, 0.075, and 0.1 for corresponding values of (T_s, δ_s) equal to (100, 6), (60, 4), (40, 3) and (20, 2), respectively. Clearly, the choice of $T_s = 100$ and $\delta_s = 6$, corresponding to A gives the lowest duty cycle of 0.06.

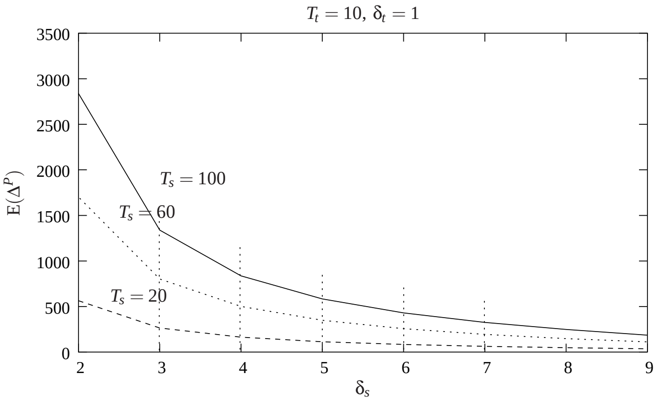

Fig. 8. $E(\Delta^P)$ vs. δ_s, $P = 0.95$.

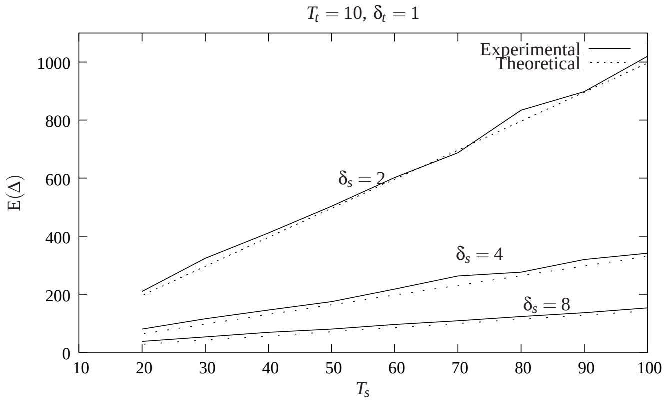

Fig. 10. $E(\Delta)$ vs. T_s.

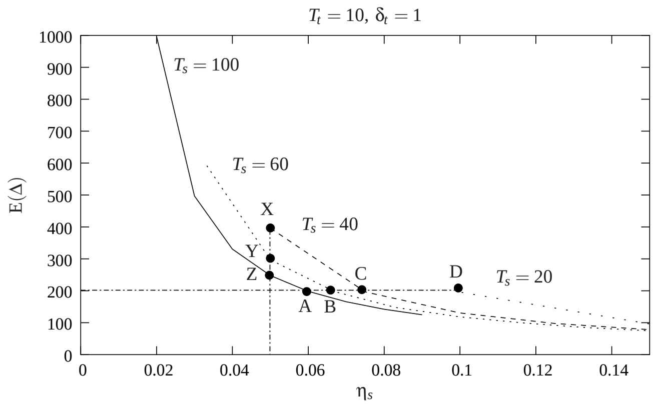

Fig. 9. $E(\Delta)$ vs. η_s.

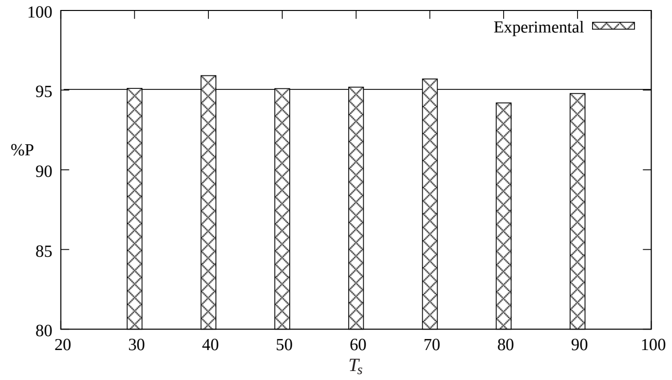

Fig. 11. Probability, P, that the target will get detected in k or fewer attempts, $P = 0.95$.

Similarly, X, Y, Z correspond to the same duty cycle of 0.05, but for different choices of T_s and δ_s. They, however, result in different expected delay of $E(\Delta)$ = 400, 300 and 250, respectively, corresponding to different choices of (T_s, δ_s) equal to $(40,2)$, $(60,3)$, and $(100,5)$. Clearly, point Z corresponding to $T_s = 100$, and $\delta_s = 5$ results in a lowest value of $E(\Delta) = 250$, but for the same η_s.

E. Validation

In this subsection we validate the theoretical performance figures given by Equations 14, and 19. To do so we simulate the system with the following parameters: $T_t = 10$, $\delta_t = 1$, and $\delta_s = 2$, T_s varies between 20 and 100 (recall that the time between two 'wake up' states of the sensor is exponentially distributed). For each T_s, we have conducted 1000 experiments and thereby computed the average delay $E(\Delta)$. This is compared in Figure 10 with the computed value predicted by Equation 14. The above simulation is repeated for different values of δ_s viz. 4 or 8 (see Figure 10). It is observed that the simulation results are in line with those given by Theorem 3.1 or Equation 14. Continuing with the above simulation, we plot in Figure 11 the values of the probability P with which a target is detected in k or fewer attempts for different values of T_s. Clearly, k is dependent on T_s, and on P, and is given by Theorem 3.2 or Equation 18. We obtain P, the observed probability, by observing how many times out of 1000 experiments did the sensor detect the target in k or fewer attempts. For $P = 0.95$, these are $k = 28$ to 29, for $T_s = 30$ to 90. The observed probability P is plotted in Figure 11. Again, the results for simulations match those predicted by Equation 18, or that the experimentally observed value of P is more or less the same as assumed value of $P = 0.95$.

IV. Periodic wake-up schedule for detection of target

In this schedule, and as before, the target sends beacons periodically, and of duration $\delta_t = \frac{N}{C}$, where N is number of bits in a beacon and C is the transmission rate of the target. However, in this new wake-up schedule, the sensor wakes up periodically. For the present, we assume that the clocks of the target and the sensor have

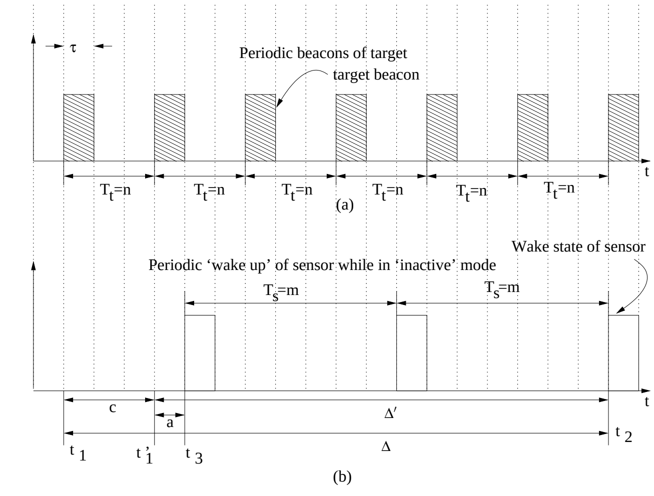

Fig. 12. Detection delay in Periodic wake-up schedule.

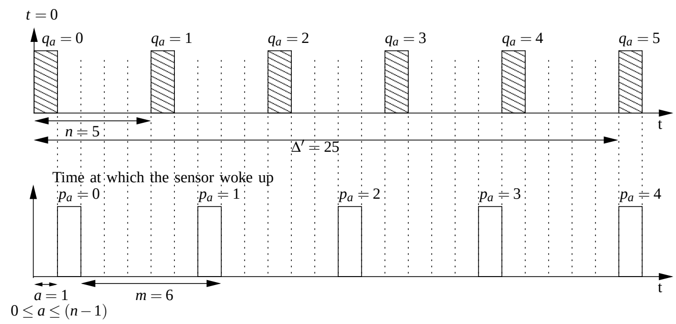

Fig. 13. q_a^{th} target beacon and p_a^{th} sensor wake-up in Periodic wake-up schedule.

the same frequency. We further assume that the target and the sensor are synchronised not only at the clock level but at the level of 'ticks', where a tick, τ, is some multiple of clock intervals, and is equal to the duration of the beacon δ_t, or that $\tau = \delta_t$ (see Figure 12).

In view of the above, the target sends beacons of duration $\delta_t = \tau$ time units with periodicity $T_t = n\tau$. A sensor wakes up for a duration $\delta_s = \tau$ time units with periodicity $T_s = m\tau$. We assume that $T_s > T_t$, or equivalently $m > n$, since a sensor is invariably starved of energy and can, therefore, wake up infrequently. Further, a sensor detects the target when the beacon and the wake-up interval overlap. Since the target and the sensor are synchronised at the level of ticks, the beacon and the sensor wake-up intervals overlap completely or do not do so at all.

To keep things simple (or without loss of generality), we assume that the time is measured in ticks of τ duration, and that $T_t = n$, $T_s = m$, and $\delta_t = \delta_s = 1$ (as in Figure 12). We define Δ (in number of ticks) as the delay in detecting the presence of the target after it appears in the radio range of the sensor. We, however, split the delay Δ, into two parts (see Figure 12).

$$\Delta = c + \Delta', \tag{23}$$

where c is

$$c = t_1' - t_1. \tag{24}$$

As before, t_1 corresponds to the time the target sends the first beacon soon after it appears within the radio range of the sensor, and t_1' corresponds to the time the target sends a beacon that is last missed (or detected in some cases) by the sensor. In particular, if t_3 is the time when the sensor wakes up for the first time after the target appears within the radio range of the sensor, then

$$t_1 \leq t_1' \leq t_3, \tag{25}$$

and t_1' is the time when the last beacon is sent, but before t_3. Now that t_1' is defined, Δ' can be re-written as

$$\Delta' = t_2 - t_1', \tag{26}$$

where t_2 is the time when the sensor has detected a beacon from the target. We now define

$$a = t_3 - t_1'. \tag{27}$$

Clearly, $0 \leq a \leq (n-1)$ (see also Figure 12).

The component c in the detection delay, Δ, can be easily shown to have a maximum value of $\left\lfloor \frac{m}{n} \right\rfloor n$, or

$$c \leq \left\lfloor \frac{m}{n} \right\rfloor n. \tag{28}$$

In order to determine a maximum (or average) value for Δ', we let $t_1' = 0$. As a result, the target sends beacons at times $0, n, 2n, \ldots, qn, \ldots$. The sensor wakes up at times $a, a+m, a+2m, \ldots, a+pm, \ldots$, where $0 \leq a \leq (n-1)$. In order to determine whether the sensor will ever wake-up so that this coincides with the beacon, we examine the question: does there exist, and under what conditions, an X such that

$$q_a n = a + p_a m, \tag{29}$$

for any given value of a, $0 \leq a \leq (n-1)$, and such that

$$q_a n \leq X, \forall a. \tag{30}$$

If there is such an X, then one can establish X as an upper bound on the delay corresponding to Δ'. This should be clear since the target will have been detected at time $q_a n = a + p_a m$. Here the beacon is sent by the target at $q_a n$, and the sensor wakes up to detect the target at $a + p_a m$.

A. Results

We now prove that the upper bound X, on the delay component, $\Delta' \le X = (m-1)n$, provided $n \le (m-1)$ and $\gcd(n,m) = 1$.

Lemma 4.1: If

$$n \le (m-1), \quad \gcd\,(n,m) = 1, \tag{31}$$

then for any and every $a, 0 \le a \le (n-1)$, $\exists p_a$, $\exists q_a$, such that

$$p_a m + a = q_a n, \tag{32}$$

where

$$0 \le q_a \le (m-1),\ 0 \le p_a \le (n-1). \tag{33}$$

Further, $\Delta' \le X = n(m-1)$, independent of $0 \le a \le (n-1)$.

Proof: Case 1: $a = 0$: Clearly, $\exists q_0 = 0$, and $\exists p_0 = 0$, such that

$$p_0 m + a = q_0 n,\ 0 \le p_0 \le (n-1);\ 0 \le q_0 \le (m-1). \tag{34}$$

Case 2: For a, $1 \le a \le (n-1)$: we need to establish that, $\exists p_a, q_a$ such that $0 \le q_a \le (m-1)$, $0 \le p_a \le (n-1)$ and

$$p_a m + a = q_a n. \tag{35}$$

From Lemma[2]4.2, since $n \le (m-1)$, $\gcd(n,m)$ = 1, $\exists q$, $1 \le q \le (m-1)$ such that

$$qn \bmod m = 1. \tag{36}$$

We define $q'_a = aq$. As a result $a \le q'_a \le a(m-1)$, and from Equation (36)

$$nq'_a \bmod m = a, \tag{37}$$

or

$$n(q'_a \bmod m) \bmod m = a. \tag{38}$$

If we define $q_a = q'_a \bmod m$, then $1 \le q_a \le (m-1)$ and, from Equation 38,

$$q_a n \bmod m = a. \tag{39}$$

That is, $\exists q_a$, $1 \le q_a \le (m-1)$ such that Equation 39 holds true. As a result $\exists p_a$

$$q_a n = p_a m + a. \tag{40}$$

[2]Lemma on multiplicative inverse (See [15])

Lemma 4.2: Let $n \in \mathcal{Z} = \{1,2,\ldots,(m-1)\}$, such that, $\gcd\,(n,m) = 1$, then $\exists q$, such that $qn \bmod m = 1$, where $1 \le q \le (m-1)$.

For sure $p_a \ge 0$. Further,

$$\begin{aligned} p_a m + a &= q_a n & (41)\\ &\le (m-1)n. & (42)\end{aligned}$$

Therefore,

$$\begin{aligned} p_a &\le \frac{(m-1)n - a}{m} & (43)\\ &\le (n-1). & (44)\end{aligned}$$

In other words, for any a, $1 \le a \le (n-1)$, $\exists q_a$, $0 \le q_a \le (m-1)$, and $\exists p_a$, $0 \le p_a \le (n-1)$ such that

$$p_a m + a = q_a n. \tag{45}$$

To summarise, for any a, $0 \le a \le (n-1)$, if $n \le (m-1)$ and $\gcd(n,m) = 1$, $\exists p_a$, $0 \le p_a \le (n-1)$, and $\exists q_a$, $0 \le q_a \le (m-1)$ such that

$$p_a m + a = q_a n. \tag{46}$$

For any given a, $0 \le a \le (n-1)$, since the delay component $\Delta' = q_a n$, and $q_a \le (m-1)$,

$$\Delta' = q_a n \le (m-1)n = X. \tag{47}$$

■

Recall that the delay in detecting the target $\Delta = c + \Delta'$, where c is as in the Equation 28, then the upper bound on Δ,

$$\Delta_{max} = \left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n. \tag{48}$$

Since time is measured in ticks of duration τ, instead $T_s = m\tau$, $T_t = n\tau$, and

$$\Delta_{max} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n \right) \tau. \tag{49}$$

Assuming that the value of a, $0 \le a \le (n-1)$ (see Equation 27) is equally probable over the range $0 \ldots (n-1)$, the average value of Δ is

$$\Delta_{avg} = \frac{\Delta_{max}}{2} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n \right) \frac{\tau}{2}. \tag{50}$$

We now summarize the "Periodic-S wake-up schedule" for target detection with synchronisation between target and sensor at the level of "ticks".

Periodic-S wake-up schedule: Let the target send beacons of duration $\delta_t = \frac{N}{C}$, and consisting of N bits at C bps, and with periodicity, $T_t = n\frac{N}{C}$. If the sensor wakes up periodically for a duration $\delta_s = \frac{N}{C}$, and with periodicity, $T_s = m\frac{N}{C}$, then

- the maximum delay in detecting the target is $\Delta_{max} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n \right) \tau = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n \right) \frac{N}{C}$,

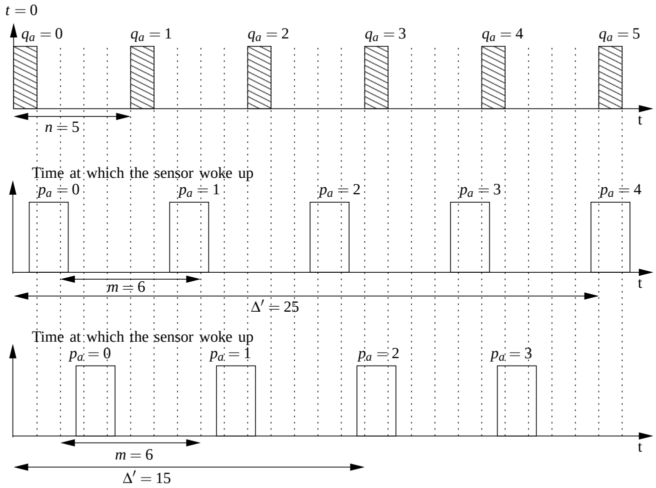

Fig. 14. Periodic wake-up (Nonsynchronised) schedule, $\delta_s = 2\delta_t$.

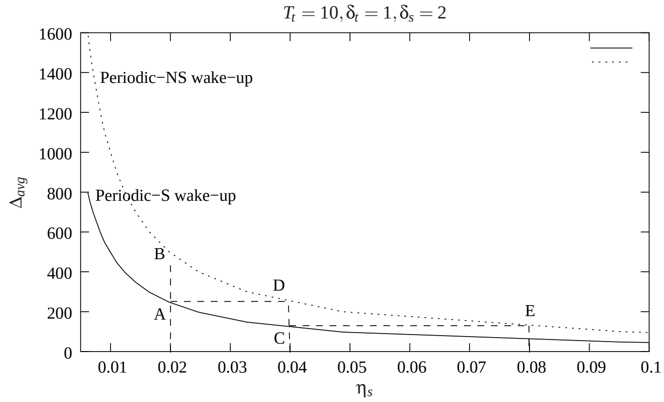

Fig. 15. Δ_{avg} vs. η_s, in Periodic-NS and Periodic-S wake-up schedules.

- the average delay, $\Delta_{avg} = \frac{\Delta_{max}}{2} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n\right) \frac{\tau}{2} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n\right) \frac{N}{2C}$, and
- the resulting duty cycle for the sensor, $\eta_s = \tau\left(\frac{1}{T_s}\right) = \left(\frac{N}{C}\right)\left(\frac{1}{T_s}\right) = \frac{1}{m}$.

Example 3: If $n = 10$, and if $m = 101$, then $n \le (m-1)$, and $\gcd(n,m) = 1$, and

- $\Delta_{max} = (10 \times 10 + 1000)\tau = 1100\tau$,
- $E(\Delta) = 550\tau$, and
- $\eta_s = 0.01$.

The schedule, viz. Periodic-S wake-up schedule described above, however, requires that the target and the sensor are synchronised thus: the beacon is always transmitted at time $t \in \{\phi_1 + i\frac{N}{C},$ for some i $, i \ge 0\}$, while the sensor wakes up at a time $t \in \{\phi_2 + j\frac{N}{C},$ for some j $, j \ge 0\}$. Synchronisation implies that the phases for the target and the sensor are the same, or $\phi_1 = \phi_2$. This can be a challenge. In order to relax this requirement of synchronisation, we simply require that the sensor wakes up for a duration $\delta_s = 2\delta_t = 2\frac{N}{C}$. To see why this is so, let the beacon be transmitted at any time[3] $t \in \{i\frac{N}{C},$ for some i $, i \ge 0\}$, while the sensor wakes up at time $t \in \{\phi_2 + j\frac{N}{C},$ for some j $, j \ge 0\}$, $0 \le \phi_2 \le \frac{N}{C}$, and remains awake till $t + 2\frac{N}{C}$. As a result the sensor is awake during time interval $[\ (j+1)\frac{N}{C}, (j+2)\frac{N}{C}\ ]$ (see also Figure 14).

We now summarize the Periodic wake-up schedule for target detection delay that does away with the requirement of synchronisation:

[3]Without loss of generality, we have set $\phi_1 = 0$

Periodic-NS wake-up schedule: Let the target send beacons of duration $\delta_s = \frac{N}{C}$, and consisting of N bits at C bps, and with periodicity, $T_t = n\frac{N}{C}$. If the sensor wakes up periodically for a duration $\delta_s = 2\frac{N}{C}$, and with periodicity, $T_s = m\frac{N}{C}$, then

- the maximum delay in detecting the target is $\Delta_{max} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n\right) \frac{N}{C}$,
- the average delay, $\Delta_{avg} = \frac{\Delta_{max}}{2} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n\right) \frac{\tau}{2} = \left(\left\lfloor \frac{m}{n} \right\rfloor n + (m-1)n\right) \frac{N}{2C}$, and
- the resulting duty cycle for the sensor, $\eta_s = 2\left(\frac{N}{C}\right)\left(\frac{1}{T_s}\right) = \frac{2}{m}$.

B. Comparison between Periodic-S wake-up and Periodic-NS wake-up schedules

A comparison of performance of the two schedules, viz. (a) Periodic-S wake-up schedule, where $\delta_s = \frac{N}{C}$, with complete synchronisation, and (b) Periodic-NS wake-up schedule, where $\delta_s = 2\frac{N}{C}$, with no requirement of synchronisation, is captured in Figure 15. Point A on the curve for Periodic-S wake-up schedule corresponds to $T_s = 51$ (or $m = 51$), resulting in $\eta_s = \frac{1}{51} = 0.02$, and $\Delta_{avg} = 275$. Point D on the curve for Periodic-NS wake-up schedule corresponds to the same $T_s = 51$ $(m = 51)$, but for which $\eta_s = \frac{2}{51} = 0.04$. The two exhibit the same performance viz. $\Delta_{avg} = 275$. Clearly, by not synchronising the sensor wake-up schedule with that of the beacon, one is forced to keep the sensor awake for twice as long, or else one is forced to live with increased delay resulting from doubling of T_s while keeping η_s fixed (represented by point B).

V. Comparison between Random wake-up and Periodic wake-up schedules

We now compare the performance of the two schedules developed above, viz.

- Random wake-up schedule, where (for instance) $T_t = 10$, $\delta_t = 1$, $\delta_s = 2$, and
- Periodic-NS wake-up schedule, where $T_t = 10$, $\delta_t = 1$ (and as a result $n = 10$), $\delta_s = 2$.

T_s is varied from 21 to 321, as a result of which $\eta_s = \frac{\delta_s}{T_s}$ varies from 0.095 to 0.006. In the case of Periodic-NS wake-up schedule the corresponding m ranges between 21 and 321. The resulting $E(\Delta)$ and $E(\Delta^P)$ (for $P = 0.95$) for Random wake-up schedule and the Δ_{max} and Δ_{avg} for Periodic-NS wake-up schedule are given in Table II, Figure 16, and Figure 17.

$T_t = 10$, $\delta_t = 1$, $\delta_s = 2$							
T_s	η_s	Periodic-NS wake-up			Random wake-up		
		(n, m)	Δ_{max}	Δ_{avg}	P	$E(\Delta^P)$	$E(\Delta)$
21	0.095	(10,21)	220	110	0.95	593	206
41	0.049	(10,41)	440	220	0.95	1162	406
81	0.0247	(10,81)	880	440	0.95	2299	806
161	0.0124	(10,161)	1760	880	0.95	4574	1606
321	0.006	(10,321)	3520	1760	0.95	9123	3206

TABLE II
Comparison Random wake-up and Periodic-NS wake-up.

T_s	δ_s	**$E(\Delta^P)$ (P=0.95)**	**$E(\Delta)$**
41	2	1162	406
61	3	815	302
81	4	677	267
121	6	521	240
161	8	399	229

TABLE III
Random wake-up, $T_t = 10$, $\delta_t = 1$, $\eta_s = 0.049$.

The values of $E(\Delta)$, or $E(\Delta^P)$ in Random wake-up schedule are significantly larger than the values of Δ_{avg}, or Δ_{max}, respectively, for the Periodic-NS wake-up schedule (see Table II). As a result, we are tempted to conclude that the Periodic wake-up schedule performs far better than the Random wake-up schedule. Their performances are indeed comparable once it is realized that the performance of the Random wake-up schedule is highly dependent on the values of δ_s. We explain this further by evaluating the performance of the Random wake-up schedule for different values of δ_s (and T_s) while keeping the duty cycle η_s constant. Table III shows values of $E(\Delta)$, and $E(\Delta^P)$ for δ_s ranging between 2 and 8, while T_s varies between 41 and 161 so as to maintain $\eta_s = 0.049$. These are also shown as points A through E in Figure 18 and Figure 19. Here $T_t = 10$, $\delta_t = 1$. δ_s is varied from 2 to 8 and T_s from 41 to 161 keeping the value of η_s constant $\eta_s = 0.049$. Table III shows that the performance of the Random wake-up schedule improves with larger values of δ_s and is comparable with the Periodic-NS wake-up schedule for the same duty cycle. In particular, for $\eta_s = 0.049$, the values $\Delta_{avg} = 220$ and $\Delta_{max} = 440$ for Periodic-NS wake-up schedule (see Table II) are somewhat comparable with $E(\Delta) = 240$ and $E(\Delta^P) = 521$ for Random wake-up schedule when $\delta_s = 6$, but for the same value of η_s. In fact, if $\delta_s = 8$ for the Random wake-up schedule, $E(\Delta^P) = 399$ is smaller than $\Delta_{max} = 440$ for the Periodic-NS wake-up schedule for the same $\eta_s = 0.049$.

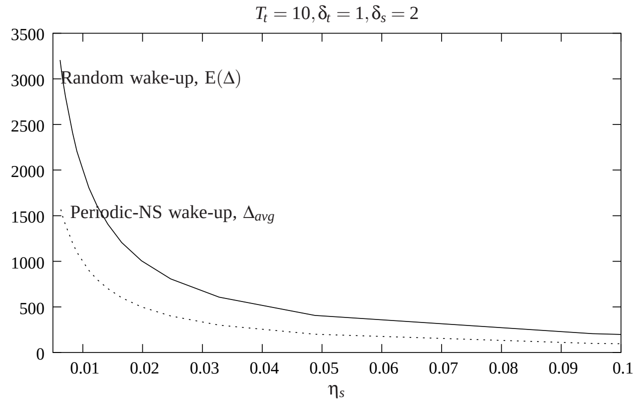

Fig. 16. Δ_{avg}, and $E(\Delta)$ vs. η_s in Periodic-NS wake-up and Random wake-up schedule.

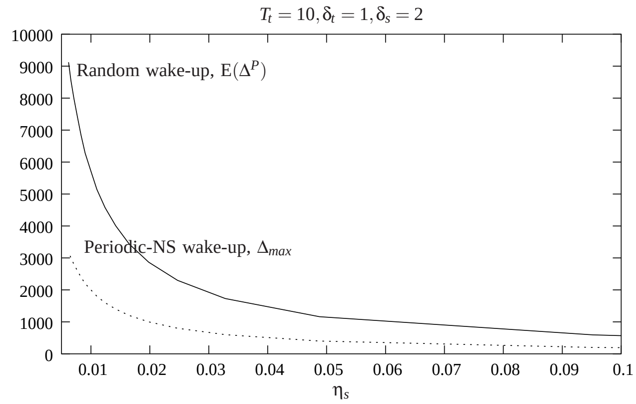

Fig. 17. Δ_{max}, and $E(\Delta^P)$ vs. η_s in Periodic-NS wake-up and Random wake-up schedule.

VI. Conclusion

We have proposed the Random and the Periodic wake-up schedules and have analysed the same from the point

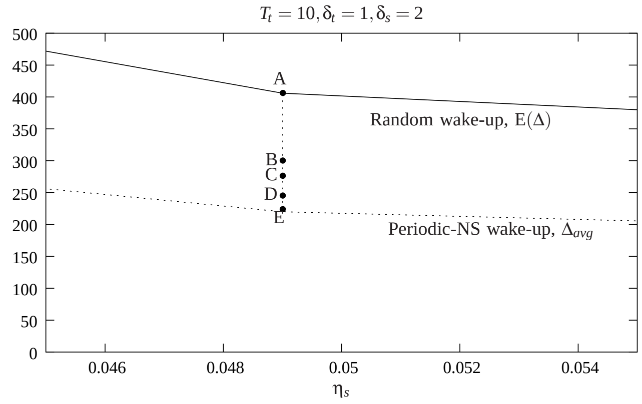

Fig. 18. Δ_{avg}, and $E(\Delta)$ vs. η_s, ($\eta_s = 0.049$) for Periodic-NS wake-up and Random wake-up schedule.

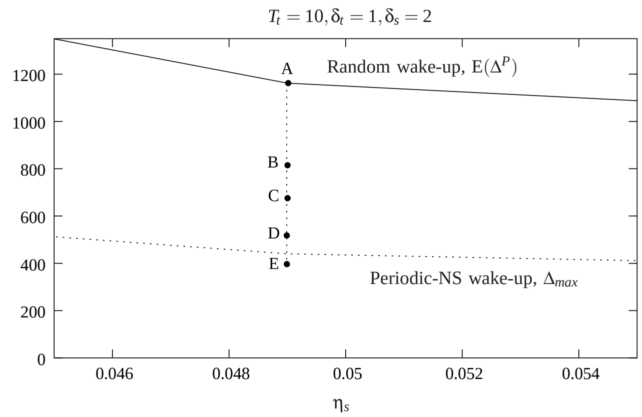

Fig. 19. Δ_{max}, and $E(\Delta^P)$ vs. η_s, ($\eta_s = 0.049$) for Periodic-NS wake-up and Random wake-up schedule.

of view of delay in detecting the target. We consider sleep schedules in which a sensor in 'inactive' mode wakes up either randomly or periodically to detect presence of the target within its vicinity resulting into two sleep schedules viz. (a) Random wake-up schedule, and (b) Periodic-NS wake-up schedule respectively. Specifically, we analyse and obtain for the Random wake-up schedule the expected delay in detection, viz. $E(\Delta)$, and the delay, $E(\Delta^P)$, such that with probability P, $\Delta \leq E(\Delta^P)$. For the Periodic-NS wake-up schedule we show that there exists an upper bound, Δ_{max}, on the delay. Further we compute the average value, Δ_{avg}, of delay. We have shown that the theoretically computed averages and the upper bounds on the delay match with the simulation results for the Random wake-up and Periodic-NS wake-up schedules.

We note that the delays for the Random wake-up schedule are significantly larger than that for the Periodic-NS wake-up schedule for smaller values of δ_s, the duration for which the sensor must be awake. The performances of the Random wake-up and the Periodic-NS wake-up schedules are comparable once it is realized that the performance of the Random wake-up schedule is highly dependent on the values of δ_s. We show that the performance of the Random wake-up schedule improves with larger values of δ_s and is comparable with the Periodic-NS wake-up schedule for the same duty cycle.

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