Problem Given n jobs $J_1, J_2 \ldots J_n$ and certain constraints $J_i < J_k$ to denote that J_i must precede J_k, we want to find a feasible scheduling of n jobs or determine that it is not possible

$$J_A \quad J_B \quad J_C \quad J_D$$

$$J_A < J_B, \quad J_A < J_D, \quad J_D < J_C, \quad J_C < J_B$$

$$\begin{tikzcd} J_A \arrow[r] \arrow[d] & J_B \\ J_D \arrow[r] & J_C \arrow[u] \end{tikzcd}$$

If the precedence graph contains a cycle $\Rightarrow$ not feasible

We want to label the vertices of the precedence graph

$f: V \to \{1, 2, 3 \ldots n\}$ s.t.

$\forall \quad J_i < J_k \quad f(i) < f(k)$

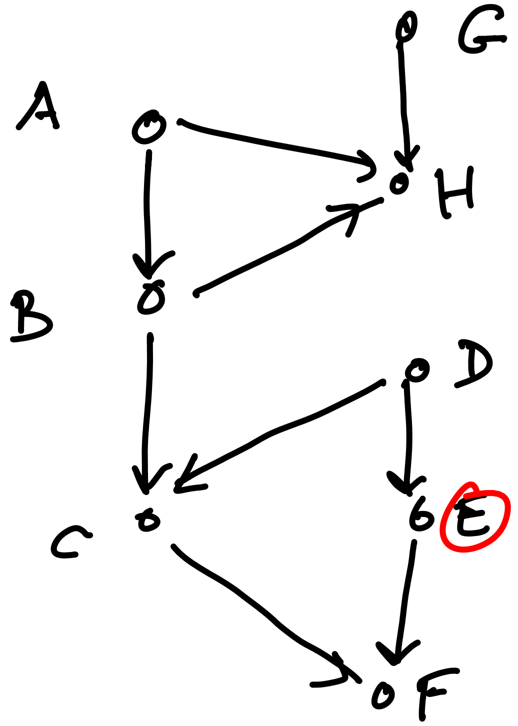
If we do not have a cycle, is it always possible?

Is it always possible to number a DAG?

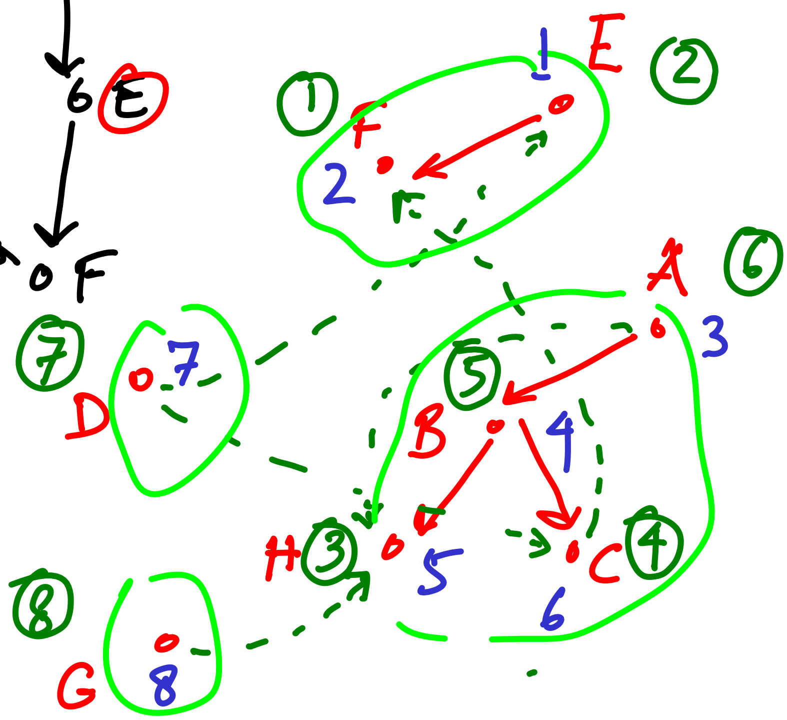
Using a simple induction on the number of vertices, and numbering a sink as n, we can accomplish this.

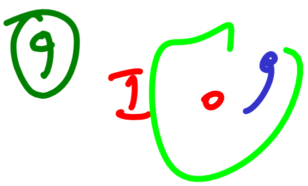
A topological sort can be done iff the directed graph has no cycles and can be done in $O(m+n)$ steps

$m = |E| \quad n = |V|$.



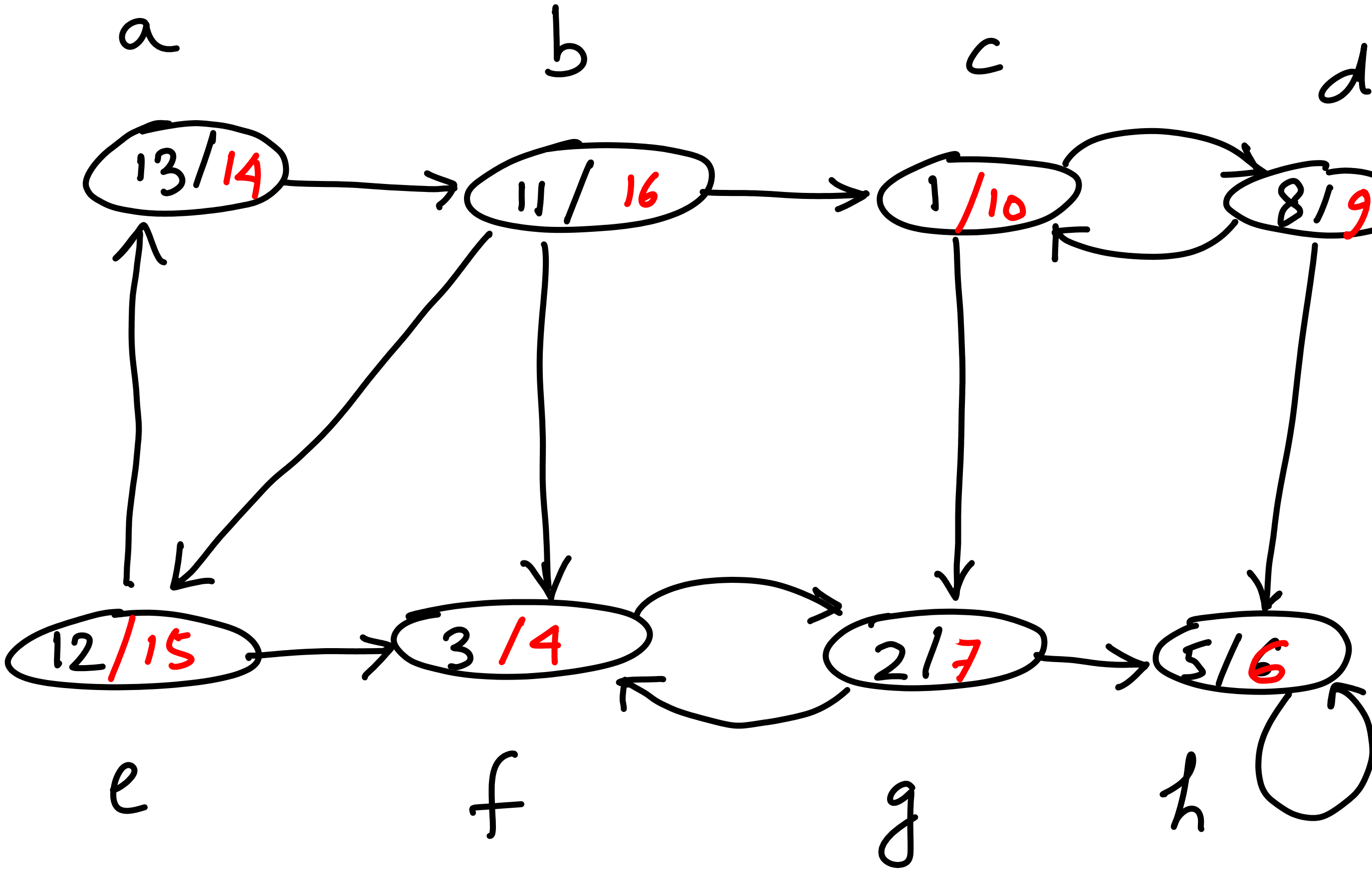
DFS numbering is a preorder numbering of the tree





How do we use preorder/postorder numbering of the DFS-tree to accomplish topological sort?

Claim: If $v \rightsquigarrow u$, then the postorder(v) > postorder(u) in a DAG.

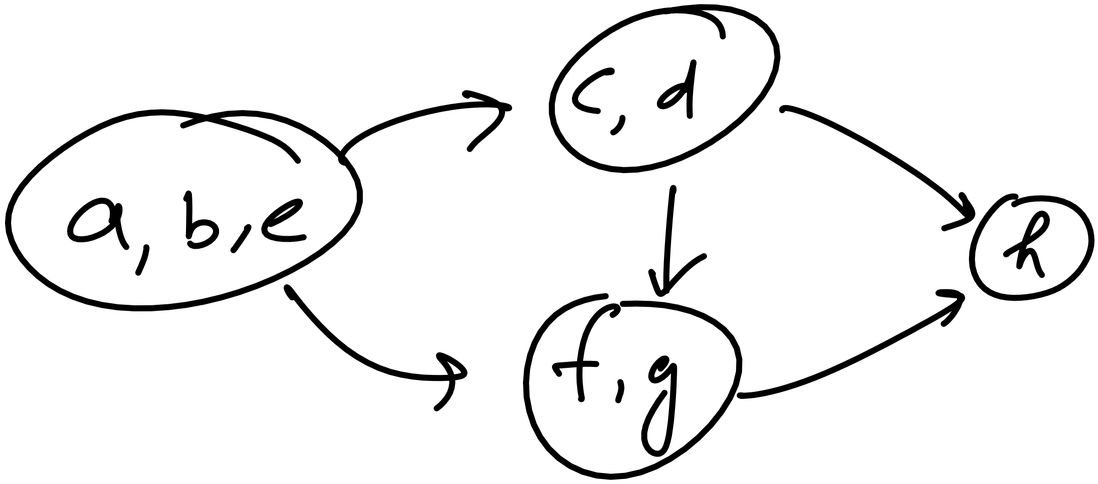


$A \rightsquigarrow B$ doesn't imply $B \rightsquigarrow A$

Suppose $A \rightsquigarrow B$ and $B \rightsquigarrow A$

If $C \rightsquigarrow A$ and $A \rightsquigarrow C$, is it true then (C, B) is related?

<u>Strongly connected Component</u> (SCC): A subset $W \subset V$ s.t. $x, y \in W$, $x \rightsquigarrow y$ and $y \rightsquigarrow x$



Component Graph is a DAG

<u>Observation</u>: The strongly connected components remain the same if we reverse the direction of every edge — call that graph G^R

(The component graph also remains same except the direction of edges)

Claim: If we do a DFS on G, then one of the vertices in a "source" component will have the largest postorder number