CSL 356 Lecture 30 Oct 11

Planar Convex hull → contd

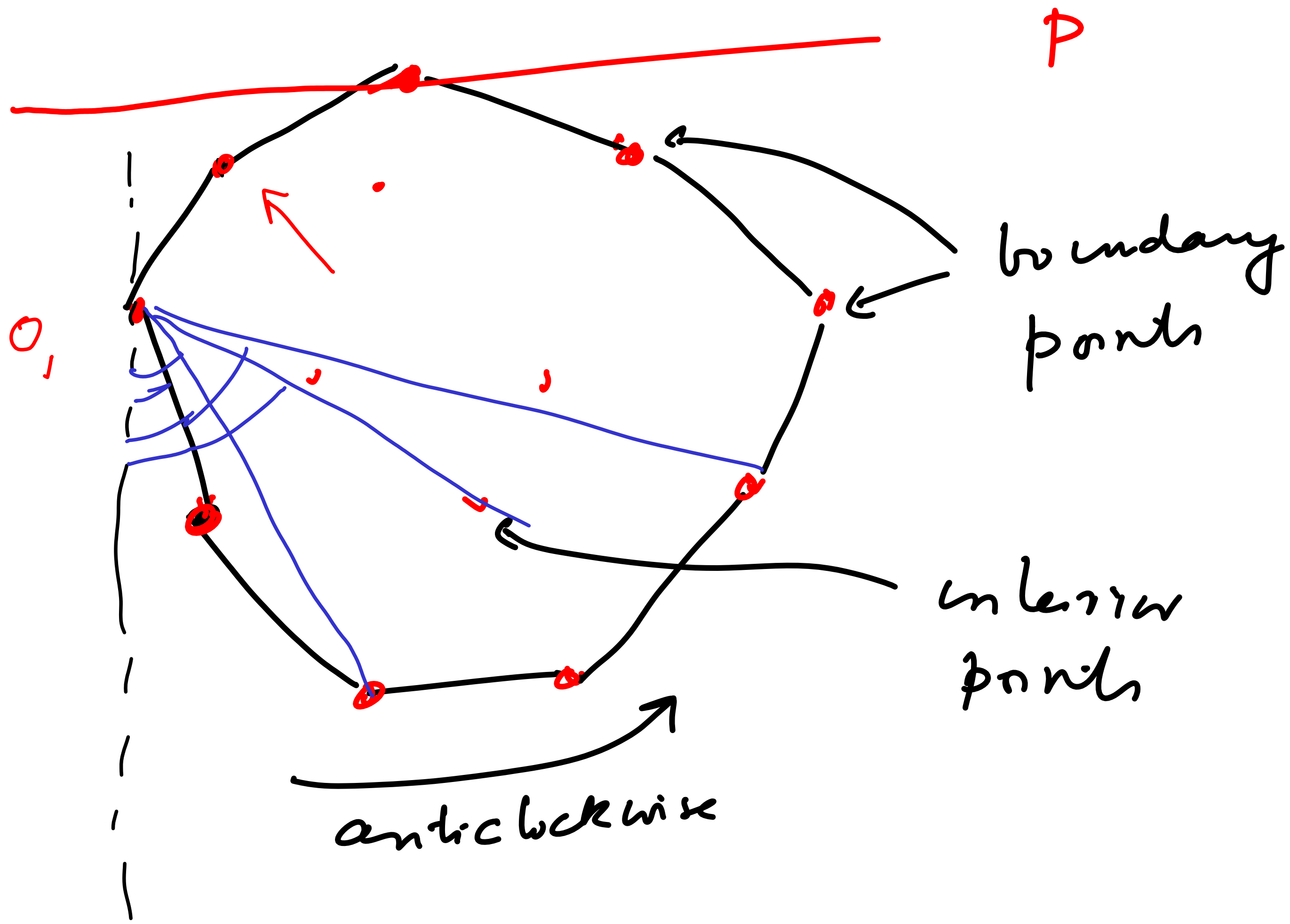
Smallest convex set containing a given set P of n points on plane
(x_1, y_1) (x_2, y_2) . . (x_n, y_n)
$CH(P)$

Some properties of convex sets

① the entire line segment joining two points p_1, p_2 in $CH(P)$ should be completely within $CH(P)$
(i.e. it must be the closure of such points)

$p_1 \cdot \alpha + (1-\alpha) p_2$: convex linear combination of p_1, p_2
$0 \leq \alpha \leq 1$

② Intersection of convex sets is convex



Pull a string around P to identify the boundary points.

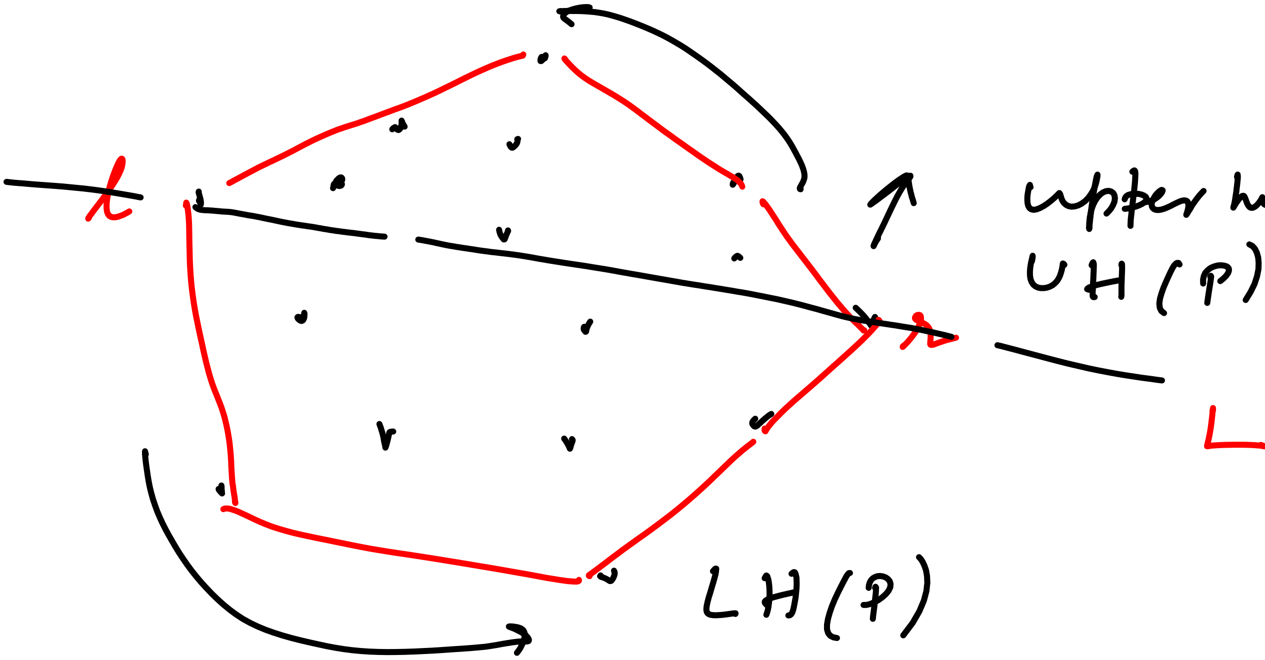
(Javis March)

Gift wrapping

To identify the next boundary point, we do $O(n)$ computation

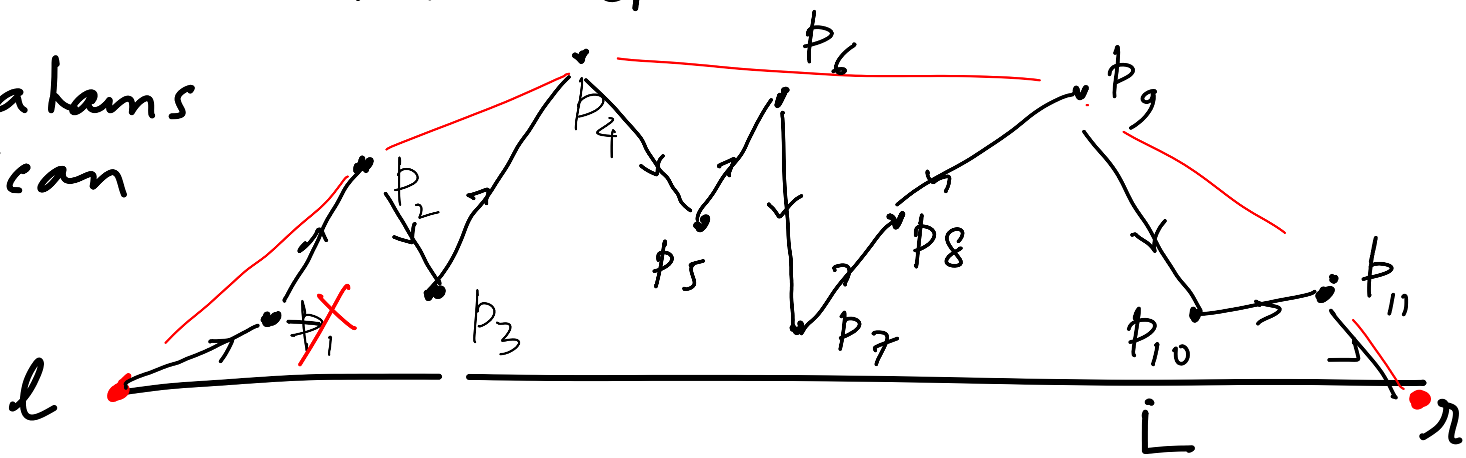
$\Rightarrow$ For h points on boundary $O(nh)$

$h \leq n$



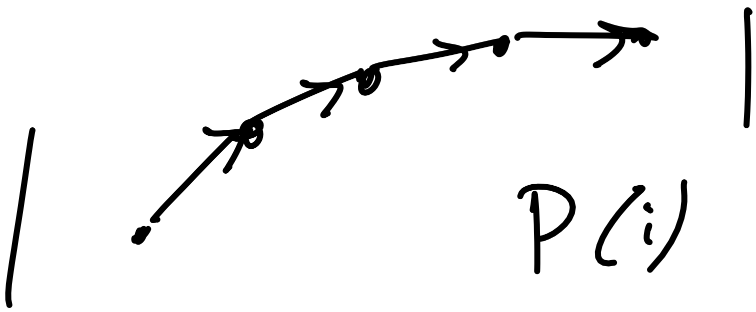
$P \cap UH(P)$ can be identified in $O(n)$ steps

Grahams Scan



Invariant: Let $P(i)$ be the boundary points among $p_1, p_2, p_3 \ldots p_i$

P(i) is a convex chain
(always turning right)



Each new point can lead to
multiple Right turn/Left turn
tests

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} < 0 \quad \text{(is right turn)}$$

If we use a stack to store P(i)
then the # of tests = # pops
from the stack

$\Rightarrow$ $O(n)$ tests overall +
time to sort $\Rightarrow O(n \log n)$