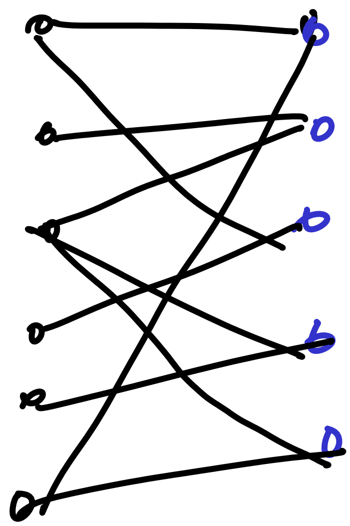
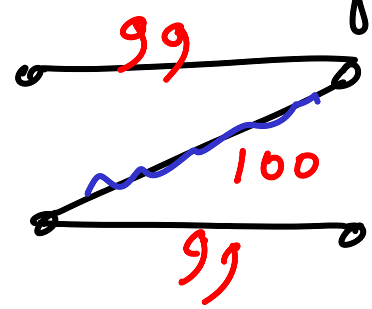
CSL 356 lecture 18, Sept 10

Path Contd after



Maximum Weighted Matching (MWM)

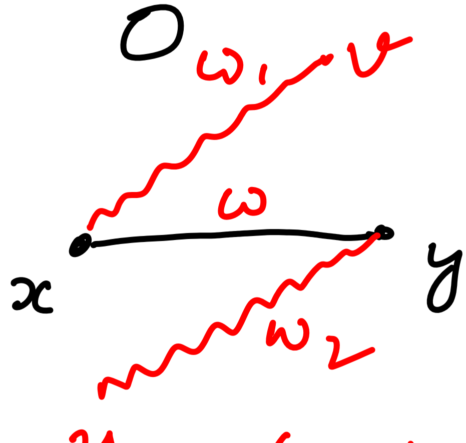
Basic greedy fails since exchange property not satisfied



but the soln output by Greedy, say G is at least $\frac{O}{2}$ where O is the optimum soln

Let O denote the set of edges that corresponds to the optimal soln

G O $G \le O$



$(x,y) \in O$

$(x,y) \in G$ $(x,y) \notin G$

$w_1 > w$
$w_2 > w$

$(x,v), (u,y) \in O$

$(x, v)\ /\ uy \in G - O$

We can can do a counting argument between edges in $G - O$ and $O - G$.

$G - O$	$O - G$
e_1	e_1'
e_2	e_2'
$\vdots$	$\vdots$
e_k	e_ℓ'

e_i "prevents" e_j' if it is the first edge to stop e_j'

$w(e_i) \geq w(e_j')$

e_i can "prevent" at most two edges in $O - G$, say e_i', e_i''.

$$\Rightarrow w(e_i) \geq \frac{w(e_i') + w(e_i'')}{2}$$

So $w(e_1) + w(e_2) + \dots w(e_k) \geq$

$$\frac{w(e_1') + w(e_1'')}{2} + w(e_2') + \dots \frac{w(e_i') + w(e_i'')}{2} + \dots$$

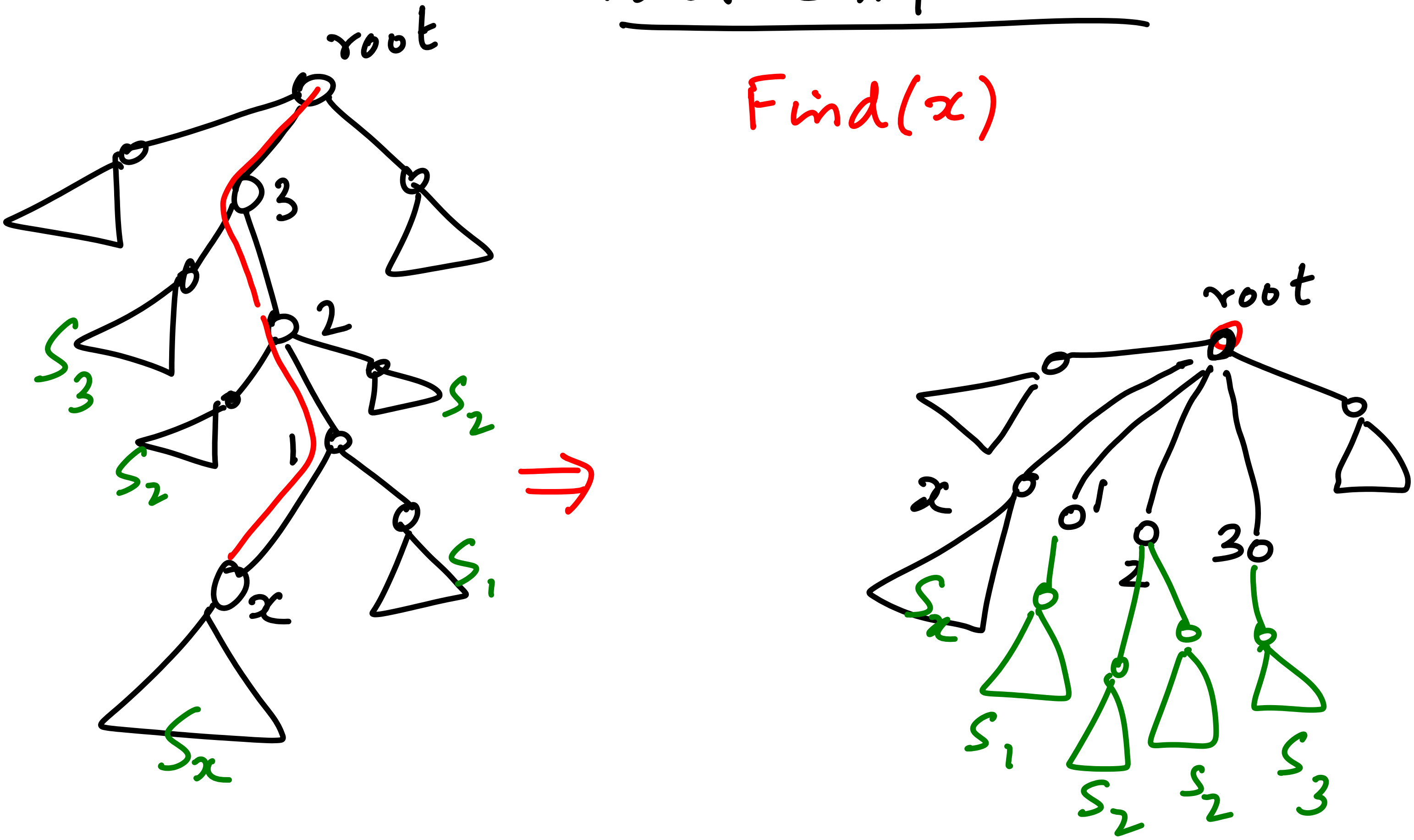
$$\geq \frac{w(O - G)}{2}$$

So $W(G - O) + W(G \cap O) \geq \frac{W(O - G)}{2} + W(G \cap O)$

$$W(G) \geq \frac{W(O - G)}{2} + \frac{W(G \cap O)}{2} = \frac{W(O)}{2}$$

Path Compression

Find(x)



All the nodes in the path $x \rightsquigarrow$ root are made direct descendents of root. Overall effect is to bring a number of nodes closer to root within the same asymptotic complexity as Find(x)

<u>Note</u>: Doesn't affect the properties on the rank

The overall cost of m Finds & n unions is $O((m+n)\log^* n)$ using the path comp & rank heuristic

$$\log^*(2) = 1$$

$$\log^* x = i \text{ if } \underbrace{\log(\log(\log}_{i \text{ times}} x)) < 2$$

Eg. $\log^*(2^{2^1}) = \log^* 2 + 1 = 2$

$\log^*(2^{2^2}) = \log^* 2 + 1 + 1 = 4$

$\log^*(2^x) = \log^* x + 1$

A part of family called inverse Ackerman function, the slowest growing function