Fast Approximate Inference in Higher Order MRF-MAP Labeling Problems

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MRF-MAP Inference Problem

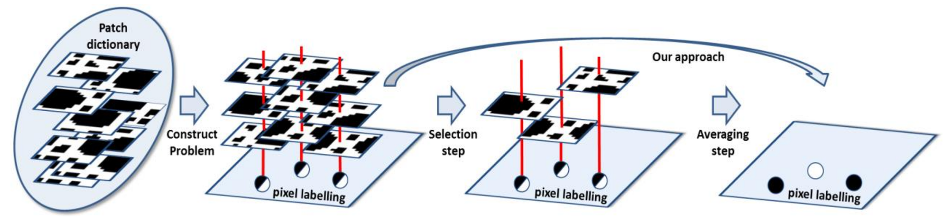
$$argmin\, E(l_p) = \sum_{p\in P} D_p(l_p) + \sum_{c\in C} W_c(l_c)$$

P is the set of pixels, C is the set of cliques.
D_p is per pixel unary/data cost.
W_c is clique prior/potential.

- Inference problem is NP hard in general

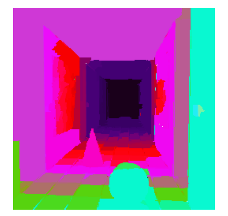
Higher Order MRF-MAP

Allows more complex clique potential based upon learnt patterns [4].

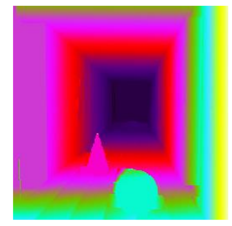


Structural constraints based upon shape and gradients can only be encoded using higher order potentials [9].

Reference Image



Disparity 2-clique



Disparity 3-clique

Inference Algorithms

Problem Type	Optimal Inference	Approximate Inference
2-Label First Order	Graph Cuts (max flow)	QPBO [3]
2-Label Higher Order	**Generic Cuts** [1]	**Proposed Algorithm**, Reduction [8]
Multi-Label First Order	Ishikawa [9]	Alpha Expansion
Multi-Label Higher Order	**MLGC** [2]	Message Passing Variants [10,11,12]

Primal

$$min_{X_p^l, Y_c^{l_c}} \sum_{p,l} D_p(l) X_p^l + \sum_{p,l_c} W_c(l_c) Y_c^{l_c}$$

s.t. $\sum_l X_p^l = 1$, $\quad \sum_{l_c: l_c^p = l} Y_c^{l_c} = X_p^l$

$X_p^l \in [0,1]$ and $Y_c^{l_c} \in [0,1]$

Dual

$$\max_p \ (U_p)$$

s.t. $U_p \le h_p^l$

where $h_p^l = D_p(l) + \sum_{c:p\in c} V_{c,p,l}$

and $\sum_{p\in c} V_{c,p,l_c^p} \le W_c(l_c)$

Approximate Cuts (AC)

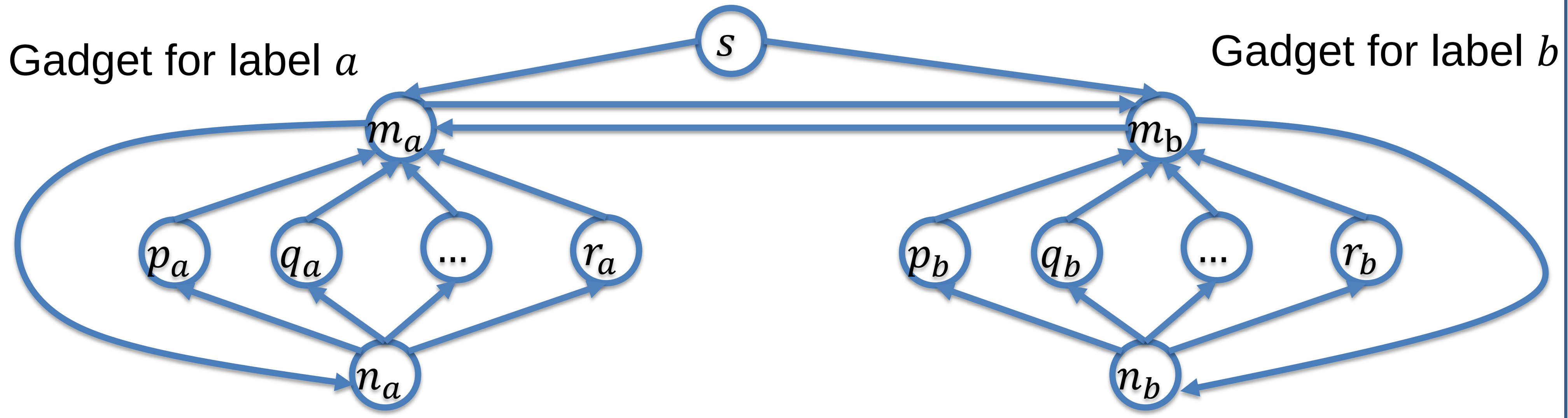
Complimentary Slackness Condition

$$Y_c^{l_c} > 0 \Rightarrow \sum_{p\in c} V_{c,p,l_c^p} = W_c(l_c)$$

Which can also be written as

$$\sum_{p\in c: l_c^p = a} V_{c,p,a} + \sum_{p\in c: l_c^p = b} V_{c,p,b} = W_c(l_c)$$

Proposed Gadget for Non-submodular Potentials



$$V_{c,p,a} = f_{n_a \to p_a} - f_{p_a \to m_a} \qquad V_{c,p,b} = f_{n_b \to p_b} - f_{p_b \to m_b}$$

Capacity Constraints:

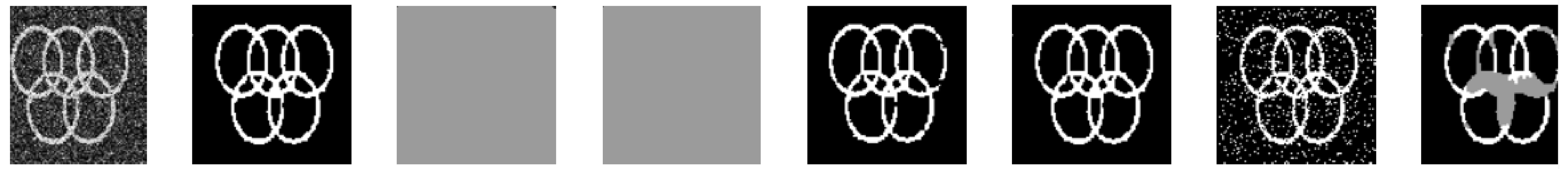
$$\sum_{p\in c: l_c^p = a} (f_{n_a \to p_a} - f_{p_a \to m_a}) + \sum_{p\in c: l_c^p = b} (f_{n_b \to p_b} - f_{p_b \to m_b}) \le W_c(l_c)$$

Weak Persistence

- We have embedded a k-ary potential function f(.) in a $2k$-ary function $g_c(\cdot,\cdot)$ such that $g_c(x, \bar{x}) = W_c(x)$ and $g(x,y) = \infty, y \ne \bar{x}$.
- The Approximate Cuts compute a submodular approximation $g^*(\cdot,\cdot)$ of $g(\cdot,\cdot)$.
- Weak Persistence is guaranteed along the lines of Kahl and Strandmark [6] and Windheuser et al. [5] .
- Node labels are guaranteed to be weakly persistent whenever the two graph nodes corresponding to a pixel are on opposite sides of cut.

Results

Denoising – 4 Clique: Submodular Potential (Optimal Inference)

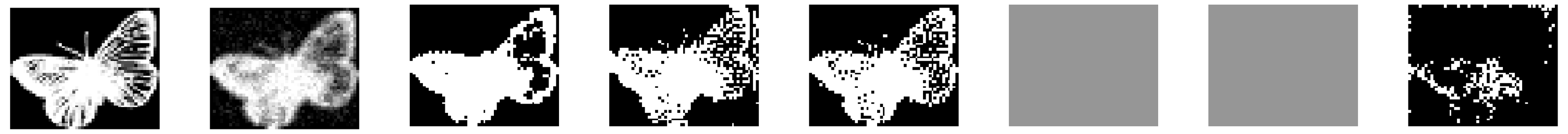


Input | AC(0.01s) 673535 673535 | IQ(0.4s) 775052 | FZ(0.2s) 775052 | DD(6.9s) 797675 | GTRWS(0.22s) 673535 673535 | MPI(0.2s) 824715 | KS(210.8s 709283 668276

Stereo – 4 Clique

GT | AC (20.5%,2.1) | DD (40.9%,4457) | GTRWS (20.7%,17.1) | FZ (62.1%,12.7) | IQ (61.7%,16.5) | MPI (72%,0.38)

Deblurring – 9 Clique



Ground Truth | Input(10) | AC (38.2%,0.115) | DD (42.18%,28.8) | GTRWS (38.8%,5.17) | FZ (99.79%,30.45) | IQ (99.79%,137.7) | MPI (49.29%,0.092)

Deblurring – 4 Clique

Input(20) | AC (12.4%,0.009) | DD (14.5%,6.3) | GTRWS (12.5%,0.079) | KS (13.9%,82.89) | FZ (18.4%,0.080) | IQ (45.8%,0.192) | MPI (20.9%,0.104)

AC runs orders of magnitude faster with superior visual quality

References

1. C. Arora, S. Banerjee, P. Kalra, and S. Maheshwari. Generic Cuts: An efficient optimal algorithm for submodular MRF-MAP problems with higher order cliques. In ECCV, 2012.
2. C. Arora and S. Maheshwari. Multi Label Generic Cuts: Optimal Inference in Multi Label Multi Clique MRF-MAP Problems. In CVPR, 2014.
3. V. Kolmogorov and C. Rother. Minimizing nonsubmodular functions with graph cuts-a review. TPAMI, 2007.
4. C. Rother, P. Kohli, W. Feng, and J. Jia. Minimizing sparse higher order energy functions of discrete variables. In CVPR, 2009.
5. T. Windheuser, H. Ishikawa, and D. Cremers. Generalized roof duality for multi-label optimization: optimal lower bounds and persistency. In ECCV, 2012.
6. **[KS]** F. Kahl and P. Strandmark. Generalized roof duality for pseudo-boolean optimization. In ICCV, 2011.
7. Woodford, O., Torr, P., Reid, I., Fitzgibbon, A.: Global stereo reconstruction under second order smoothness priors. In CVPR, 2008.
8. **[IQ]** Ishikawa, H.: Transformation of general binary MRF minimization to the first-order case. In TPAMI 2011.
9. H. Ishikawa. Exact Optimization for Markov Random Fields with Convex Priors. In TPAMI 2003.
10. **[DD]** N. Komodakis and N. Paragios. Beyond pairwise energies: Efficient optimization for higher-order MRFs. In CVPR, 2009.
11. **[MPI]** D. Koller and N. Friedman. Probabilistic Graphical Models: Principles and Techniques. Adaptive Computation and Machine Learning. MIT Press, 2009.
12. **[GTRWS]** V. Kolmogorov. Reweighted message passing revisited. CoRR, abs/1309.5655, 2013.
13. **[FZ]** A. Fix, A. Gruber, E. Boros, and R. Zabih. A graph cut algorithm for higher-order markov random fields. In ICCV, 2011