CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 9: Graph Theory: Paths and Cycles

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1. (**Diestel: Proposition 1.3.1**) Every graph G contains a path of length $\delta(G)$ and a cycle of length at least $\delta(G)+1$, provided $\delta(G)$, the minimum degree of the graph, is at least 2.

2. (**Diestel: Proposition 1.3.2**) Every graph G containing a cycle satisfies $g(G) \leq 2 \cdot \mathrm{diam}(G) + 1$, where $g(G)$ is the girth of the graph i.e. the length of the smallest cycle in the graph, and $\mathrm{diam}(G)$ is the diameter of the graph.

3. (**Diestel: Chap 1, Prob 3**) Let G be a graph containing a cycle C, and assume that G contains a path of length at least k between some two vertices of C. Show that G contains a cycle of length at least $\sqrt{k}$. Is this the best possible?