CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 6: Counting

Instructor: Amitabha Bagchi

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1. Argue in words with no mathematical manipulation whatsoever:
   (a) $\binom{n}{k} = \binom{n}{n-k}$
   (b) $\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}$
   (c) $\binom{n}{k}\binom{n-k}{j} = \binom{n}{j}\binom{n-j}{k}$
   (d) $\sum_{i=0}^{i=n}(-1)^i\binom{n}{i} = 0$
2. How many ways of giving k apples to n children such that each child gets at least one? Approximately what fraction of the total number of ways of giving k apples to n children involves each one of them getting at least one? You can use the following inequalities:

$$\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

3. How many ways of giving k apples to n children such that there is a child who gets at least two apples? How many ways in which there is a child who gets exactly two apples? Which is larger?
4. A particular child, Chunnu, doesn't like apples. Every time he gets an apple he gives it to one of his friend. He always gives the first apple he gets to Aakanksha, the next to Abhay, the third to Achyut, the fourth to Aditi and so on down the alphabetical order. How many ways are there of giving k apples to these n children?