CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 4: Predicate logic etc.

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1. A certain country is inhabited only by people who either always tell the truth or always tell lies and who will respond to questions only with a "yes" or a "no." A tourist comes to an unmarked fork in the road where one branch leads to the capital and the other does not. There is an inhabitant, Mr Z., standing at the fork. What *single* question should the tourist ask him to determine which branch to take?

2. For what positive integers x is the statement $(x-2)^2+1 \leq 2$ true? For what integers is it true? For what real numbers is it true? If we expand the universe for which we are considering a statement about a variable, does this always increase the size of the statment's truth set?

3. Using $s(x, y, z)$ to be the statement $x = yz$ and $t(x, y)$ to be the statement $x < y$, write a formal statement for the definition of the greatest common divisor of two numbers.

4. Rewrite the following statement without any negations: It is not the case that there exists an integer n such that $n > 0$ and for all integers $m > n$, for every polynomial equation $p(x) = 0$ of degree m there are no real numbers for solutions.