CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 2: Sets/Mathematical Induction

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1. Prove that $|A| < |2^A|$ for all sets $|A|$. Argue that this statement is equivalent to proving that there cannot exist a surjection from A to 2^A and then prove that no such surjection exists.
2. Show that any infinite subset of $\mathbb{N}$ is countable.
3. Prove that every number greater than 7 is a sum of a nonnegative integer multiple of 3 and a nonnegative integer multiple of 5.
4. Prove by induction:
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n}$$
5. n couples arrived at a party and were greeted by the host and the hostess at the door. After several rounds of handshaking the host asked the guests as well as the hostess (his wife) to indicate the number of hands each one of them had shaken. He got $2n+1$ different answers. Given that no one shook hands with his or her own spouse, how many hands had the hostess shaken? Prove your answer by induction.