CS105L: Discrete Structures
I semester, 2006-07

Tutorial Sheet 10: Graph Theory continued

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1. Show that every 2-connected graph contains a cycle.

2. Determine $\kappa(G)$ (vertex connectivity) and $\lambda(G)$ (edge connectivity) for P^k (a path on k vertices), C^k (a cycle with k vertices), K^k (a complete graph on k vertices), $K_{m,n}$ (a complete bipartite graph with m vertices on one side and n vertices on the other side.)

3. A connected acyclic graph is called a *tree*. Prove that the following assertions are true for a graph T.

   (a) T is a tree.

   (b) any two vertices of T are linked by a unique path in T.

   (c) T is minimally connected i.e. T is connected but $T - e$ is disconnected for every edge $e \in T$.

   (d) T is maximally acyclic i.e. T contains no cycles but $T + xy$ does for any two non-adjacent vertices $x, y \in T$.

4. An *independent set* in a graph G is a set of vertices which induce a subgraph on G which has no edges. The *chromatic number* of G, denoted $\chi(G)$, is the minimum number of independent sets which cover the entire graph i.e. the minimum number of independent sets whose union is the entire vertex set, V, of G. If d is the maximum degree of the graph show that
$$\chi(G) \leq d + 1.$$