CS105L: Discrete Structures
I semester, 2006-07

Homework # 9

Due before class on **Friday, October 20th, 2006**

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1. Recall the definition of the d-dimensional cube from the previous homework: For some natural number d, let's say the vertex set of a graph is labelled with the strings from $\{0,1\}^d$ i.e. each vertex has a unique label which is a d-bit string and every d-bit string corresponds to a vertex. Further we say that there's an edge between two vertices if their labels differ in exactly one position.

   Determine the connectivity of the d-dimensional cube.

2. A *vertex colouring* of a graph is an assignment of colours (or just natural numbers) to the vertices of a graph. A *colour class* in a coloured graph is a maximal set of vertices which have been assigned the same colour. A graph is said to be (k,d)*-colourable* if the vertices can be coloured with k colours in such a way that the vertices in each colour class induce a graph of maximum degree d. Prove the following theorem due to Lovász:

   Theorem 8.1 *(***Lovász, 1966***) For any k, any graph of maximum degree Δ with $|E|$ edges can be $(k, \lfloor \Delta/k \rfloor)$-coloured in time $O(\Delta|E|)$.*