CS105L: Discrete Structures
I semester, 2006-07

Homework # 3

Due before class on **Friday, August 18, 2006**

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1. Consider the following formula, A, of the propositional calculus:

$$p \Rightarrow q \Rightarrow r \vee q \equiv p \wedge q \Rightarrow \neg r \vee q$$

   (a) Using the simple grammar defined for the propositional calculus in class, give three different derivations for A.

   (b) Do your three different dervations differ in at least one subformula from each other? Take one of your derivations and list all the subformulas for it.

   (c) For the derivation you chose in the previous part list all the interpretations in which $v(A)$ is true.

   (d) Now, chose a derivation in which $p \Rightarrow q$ is a subformula. Is $A\{p \Rightarrow q \leftarrow q \Rightarrow p\}$ logically equivalent to A? If not, give an interpretation in which the two differ. Is $A\{p \Rightarrow q \leftarrow \neg q \Rightarrow \neg p\}$ logically equivalent to A?

2. Prove the following in the Hilbert system using only the axioms, modus ponens and the deduction rule:

   (a) $\vdash (A \Rightarrow B) \Rightarrow [(B \Rightarrow C) \Rightarrow (A \Rightarrow C)]$

   (b) $\vdash [A \Rightarrow (B \Rightarrow C)] \Rightarrow [B \Rightarrow (A \Rightarrow C)]$

3. Using all the deduction rules (and axioms and MP) prove in the Hilbert system that

$$\vdash (A \Rightarrow \neg A) \Rightarrow \neg A$$