CS105L: Discrete Structures
I semester, 2006-07

Homework # 2

Due before class on **Friday, August 11, 2006**

Instructor: Amitabha Bagchi

August 3, 2006

1. Given two sets A and B, show that if there exists an injective function $f : A \rightarrow B$ and an injective function $g : B \rightarrow A$, there exists a bijection $h : A \rightarrow B$.

2. (a) If $A = \{i \in \mathbb{N} | i \leq m\}$ for some finite m. Show that the set $\mathcal{F}$ of all functions from A to $\mathbb{N}$ is countable.

   (b) Is the set of all functions from $\mathbb{N}$ to $\mathbb{N}$ countable?

3. Show that the countable union of countable sets is countable.

4. Prove that the set of all decimal fractions is uncountable.

5. A *0-2 binary tree* is a tree in which each node has 0 or 2 children. How many leaves does such a tree have? Prove your answer using induction.