CS105L: Discrete Structures
I semester, 2006-07

Homework # 10

Due before class on **Friday, November 10th, 2006**

Instructor: Amitabha Bagchi

November 2, 2006

1. Derive Hall's marriage theorem for bipartite matching from Tutte's theorem on matchings in general graphs.

2. Let M be a matching in a bipartite graph G. Show that if M contains fewer edges than some other matching M' in G then there exists an augmenting path with respect to M. Does this fact extend to matchings in non-bipartite graphs?

3. Let A be a finite set with subsets $A_1, \ldots, A_n$ and let $d_1, \ldots, d_n$ be n natural numbers. Show that there are disjoint subsets $D_k \subseteq A_k$, with $|D_k| = d_k$ for all $k \leq n$ if and only if

$$\left| \bigcup_{i \in I} A_i \right| \geq \sum_{i \in I} d_i$$

for all $I \subseteq \{1, \ldots, n\}$.