COL202: Discrete Mathematical Structures. I semester, 2017-18.
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Tutorial Sheet 6: Elementary probability
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Important: The boxed question is to be submitted at the beginning of class on a plain sheet of paper with your name, entry number and the tutorial sheet number clearly written at the top of the sheet.

Problem 1
We call a coin *fair* if Pr(Heads) = PrTails = 1/2. We call the toss of a set of coins *independent* if the probability of the outcomes on any set of coins is simply the product of the probability of the individual outcomes.

Problem 1.1
What is the probability of getting exactly three heads when you flip a fair coin five times independently?

Problem 1.2
What is the probability of getting at least three heads when you flip a fair coin five times independently?

Problem 1.3 *
Argue *rigorously* but *without calculating any probability* that the probability of getting at least three heads when you flip five fair coins independently is $\leq$ the probability of getting at least three heads when you flip ten coins independently.

Problem 2
Suppose that we have a common dice, i.e., it has 6 sides with the numbers 1 to 6 on it. We call such a dice fair (or non-Shakuni (NS) for Mahabharat lovers) if the probability of any of those numbers coming up when we throw it is equal.

Problem 2.1
What is the probability that the sum of numbers of a pair of NS dice thrown independently is at most 4?

Problem 2.2
Clearly a pair of NS dice can turn up sums of 2 to 12. Which value has the maximum probability of appearing as a sum of two such dice thrown independently?

Problem 2.3 *
Throw one NS dice. Let's say it shows i. Now throw the other dice. If it shows any value $< i$ then throw it again. Keep throwing it till it shows a value $\geq i$. Then add the two numbers. How do the values of the probabilities of the various sums (2 to 12) change? What happens if you have to throw again if the value is $\leq i$?

Problem 3
In the following we are given an urn (a *ghada* in Hindi) that contains r red balls and b blue balls. We are allowed to put our hand into the urn and pull out a ball but we can see what we are picking so let us assume that every time we pick a ball out of the urn, each ball in the urn at that time has equal probability of being picked. After taking the ball out we may put it back in (replacement) or not or we may even add some new balls into the urn.

Problem 3.1
Pick k balls, replacing each ball before picking the next one. What is the probability that.

1. All balls picked are red.
2. The first ball picked and the last ball picked have the same colour.
3. The number of red balls is equal to the number of blue balls.

Problem 3.2
Solve all subparts of Problem 3.1 assuming that the ball removed at each step is *not* replaced. You may assume, of course, that $k \leq r + b$.

Problem 3.3 *
In this problem we pick a ball then replace it in the urn *adding* one more ball of the same colour, e.g., if the ball picked is red we put two red balls back in the urn thereby increasing the number of balls in the urn. To track this process, looking at the numbers of balls is not as relevant as looking at the fraction of balls of each colour. What happens to this fraction as the number of picks tends to ∞. Note: this problem is actually outside the scope of this course and is not meant for you to solve. Figure out whatever you can about it using what you know and later look it up on the internet if you are interested.