COL202: Discrete Mathematical Structures. I semester, 2017-18.
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Tutorial Sheet 2: Recurrences
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Important: The boxed question is to be submitted at the beginning of class on a plain sheet of paper with your name, entry number and the tutorial sheet number clearly written at the top of the sheet.
Note: Starred questions may be somewhat time consuming.

Q1.1. In the 2-dimensional plane we have n lines such that no two lines are parallel and no three lines intersect at one point. If R_n is the number of regions created by these n lines, find a recurrence for R_n and solve it.

Q1.2.* Write down the analogue of this problem in a d-dimensional space, i.e., what are the conditions required on the set of $d-1$-dimensional hyperplaces for the question to yield a clean recurrence. What is the recurrence? What is its solution? How do the number of regions grow with dimension d?

Q2. Find a recurrence relation for the number of bit strings of length n that contain the string 01.

Q3.1. Let A_n be the $n \times n$ matrix with 2's on its main diagonal, 1's in all positions next to a diagonal element, and 0's everywhere else. Find a recurrence relation for d_n, the determinant of A_n . Solve this recurrence relation to find a formula for d_n.

Q3.2. Solve the recurrence obtained in Q3.1.

Q4.1. Find a recurrence relation for the number of ways to partition n elements into r non-empty groups? Note that this is a two parameter recurrence.

Q4.2.* Solve the recurrence obtained in Q4.1.

Q5. Solve the following recurrence relations:

a. a_r^2 - $2a_{r-1} = 0$, $a_0 = 4$.

b. $a_r = \sqrt{a_{r-1} + \sqrt{a_{r-2} + ...}}$, $a_0 = 4$.

c. $a_{2k} = 2a_k + 2\text{k}$

d. $a_k = a_{k-1} + a_{k-2}$, $a_0 = 0$, $a_1 = 1$

e. $a_k = 4a_{k-1}$ - $4a_{k-2}$, $a_0 = 1$, $a_1 = 3$

Q6.* Recall that in class we discussed the following problem: Given a set of n disks of equal weight and diameter 1 we want to stack them in such a way that their *spread*, i.e., the distance from the center of mass to the rightmost edge of any disk is maximised. We saw that the textbook [LLM10] suggested that to find a recurrence for this we need to look at two cases: the first being where the bottom disk is the rightmost and the second case being where the rightmost disk is not the bottom disk. While it is clear that their approach is correct since all possibilities are covered we want to explore how the authors of [LLM10] came to the conclusion that this was the right way to approach the problem. Either construct an argument that leads to the authors approach or come up with an alternate approach to the problem.

7. Recall that the Fibonacci numbers $\{f_n\}_{n\geq 0}$ are those that satisfy the recurrence $a_n = a_{n-1} + a_{n-2}$, $a_0 = 0, a_1 = 1$. Now consider the case where $a_0 = s$ and $a_1 = t$, where s and t are constants, then show that $a_n = sf_{n-1} + tf_n$ for all positive integers n.