COL202: Discrete Mathematical Structures. I semester, 2017-18.
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Tutorial Sheet 13: Logic.
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Important: The boxed question is to be submitted at the beginning of class on a plain sheet of paper with your name, entry number and the tutorial sheet number clearly written at the top of the sheet.

Note: Please read *all* of Chapter 3 of [1] (even the parts not discussed in class) before attempting this sheet. All questions in this sheet are from that book, question numbers and page indicated in brackets refer to the pdf linked on the course page.

Problem 1 (Prob 8, pp 94)
Use a truth table to show that $(s \vee t) \wedge (u \vee v)$ is equivalent to $(s \wedge u) \vee (s \wedge v) \vee (t \wedge u) \vee (t \wedge v)$.

Problem 2 (Prob 12, pp 94)
Find a statement involving only $\vee$, $\wedge$ and $\neg$ that is equivalent to $s \Leftrightarrow t$. Try to ensure that your statement has the fewest possible symbols.

Problem 3 (Prob 4, pp 106)
The definition of a prime number is that it is an integer greater than 1 whose only positive integer factors are itself and 1. Find two ways to write this definition so that all quantifiers are explicit. (It may be convenient to introduce a variable to stand for the number and perhaps a variable or some variables for its factors.)

Problem 4 (Prob 6, pp 106)
Using $s(x, y, z)$ to be the statement $x = yz$ and $t(x, y)$ to be the statement $x < y$, write a formal statement for the definition of the greatest common divisor of two numbers.

Problem 5 (Prob 10, pp 107)
Rewrite the following statement without any negations. It is not the case that there exists an integer n such that $n > 0$ and for all integers $m > n$, for every polynomial equation $p(x) = 0$ of degree m there are no real numbers for solutions.

Problem 6 (Prob 11, pp 107)
Consider the following slight modification of Theorem 3.2. For each part below, either prove that it is true or give a counterexample. Let U_1 be a universe, and let U_2 be another universe with $U_1 \subseteq U_2$. Suppose that $q(x)$ is a statement such that $U_1 = \{x \mid q(x) \text{ is true}\}$.

1. $\forall x \in U_1(p(x))$ is equivalent to $\forall x \in U_2(q(x) \wedge p(x))$.
2. $\exists x \in U_1(p(x))$ is equivalent to $\exists x \in U_2(q(x) \Rightarrow p(x))$.

Problem 7
Each expression below represents a statement about the integers. Using $p(x)$ for "x is prime," $q(x, y)$ for "$x = y^2$," $r(x, y)$ for "$x \leq y$," $s(x, y, z)$ for "$z = xy$," and $t(x, y)$ for "$x = y$," determine which expressions represent true statements and which represent false statements.

1. $\forall x \in Z(\exists y \in Z(q(x, y) \vee p(x)))$.
2. $\forall x \in Z(\forall y \in Z(s(x, x, y) \Leftrightarrow q(x, y)))$.

3. $\forall y \in Z(\exists x \in Z(q(y, x)))$.
4. $\exists z \in Z(\exists x \in Z(\exists y \in Z(p(x) \wedge p(y) \wedge \neg t(x, y))))$.

Problem 8 *

On page 114 of [1] there is a list of 12 rules of inference that have been discussed in the book. For each of the following proofs from Diestel, Chap 1, rewrite the proof as a sequence of statements in terms of predicates and quantifiers and inference rules. Your solution should begin by defining all your predicate names. After that each line should be a logical statement with a line number. If that statement has been derived from earlier statements using an inference rule then this should be mentioned with the line. *Use common sense to prevent the solutions from becoming unduly long. The purpose of this exercise is to help you understand the working of rules of inference in the real world, not to kill trees.*

1. Proposition 1.3.1.
2. Proposition 1.4.1.
3. Proposition 1.6.1.

References

[1] K. Bogart, S. Drysdale, C. Stein Discrete Math for Computer Science Students. 2005.