COL202: Discrete Mathematical Sturctures. I semester, 2017-18.
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Tutorial Sheet 1: Counting
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Important: The boxed question is to be submitted at the beginning of class on a plain sheet of paper with your name, entry number and the tutorial sheet number clearly written at the top of the sheet.
Note: Starred questions may be somewhat time consuming.

Q1.1. How many ways are there of seating n people around a circular table? Can you argue the answer in more than one way?

Q1.2. Solve Q1.1 by creating blocks of permutations of n people such that each block corresponds to one circular seating pattern, i.e., by creating a bijection between the blocks you have created and the set whose size you want to count.

Q1.3. Given a plane with integer points of the type (x, y) where both x and y are integers, we define a *lattice path* from (x_1, y_1) to (x_2, y_2) to be a set of line segments that go from a point (i, j) to $(i+1, j)$ or $(i, j+1)$, i.e., all steps in the path either move right or up.

Q1.3.1. Does a lattice path exist between any two sets of integer points on the plane?

Q1.3.2. Argue that the length of every lattice path between a pair of points the same. What is the length of the lattice path joining $(0, 0)$ to (m, n)?

Q1.3.3. How many lattice paths between $(0, 0)$ and (m, n)?

Q1.4.* A lattice path from $(0, 0)$ to (n, n) is called a *Catalan path* if it only visits points (x, y) such that $y \leq x$.

Q1.4.1. Argue that every lattice path that is *not* a Catalan path must touch or cross the line $y = x+1$.

Q1.4.2. Find a bijection between the set of lattice paths that touch or cross the line $y = x + 1$ and the set of lattice paths between (-1,1) and (n, n).

Q1.4.3. Use the arguments developed in the previous parts of this problem to give a formula for the number of Catalan paths between (0,0) and (n, n).

Q1.5. Prove the following identities regarding binomial coefficients by making counting arguments. Give as many different arguments as possible

Q1.5.1.

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}.$$

Q1.5.2.

$$\binom{n}{k}\binom{n-k}{j} = \binom{n}{j}\binom{n-j}{k}.$$

Q1.5.3.

$$\sum_{i=0}^{k}\binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}.$$