COL 7004 October 6, 7, 9

TUTORIAL SHEET 9

1. While constructing a skip list, you to promote an element to the next level with probability p and calculate the best value of p for which the product $E_q \times E_s$ is minimized where E_q, E_s are the expected query and expected space respectively. What is a suitable value of p?

2. In class, we showed that it is extremely unlikely that the height of a skip list exceeds $2 \log n$, where n is the number of keys. Show that the expected height of a skip list is $O(\log n)$.

3. Consider the following family of hash functions $\mathcal{H}$ from $[2^u]$ to $[2^m]$. Note that $[2^x]$ refers to all bit strings of length 2^x. Each hash function $h \in \mathcal{H}$ corresponds to an $m \times u$ binary matrix M, and $h(x) := Mx \bmod 2$. Show that this family of hash functions is universal.

4. A collection $X_1, \ldots, X_k$ of random variables (on the same probability space) is said to be pair-wise independent if for every pair of real values a, b and for every pair of distinct random variables X_i, X_j,

$$\Pr[X_i = a \wedge X_j = b] = \Pr[X_i = a] \cdot \Pr[X_j = b].$$

   Suppose we toss two fair coins independently, and define random variables X, Y, Z as follows: $X = 1$ if the first coin toss outcome is Heads, 0 otherwise. Similarly, $Y = 1$ if the second coin toss outcome is Heads, 0 otherwise. Finally, $Z = 1$ if the outcomes of the two coin tosses are different, 0 otherwise. Show that X, Y and Z are pair-wise independent, but they are not independent random variables.

5. Let v be a vector in $\Re^2$ with coordinates $(3, 2)$ in the standard basis. What will its coordinates be in the basis $\{(1, 1), (1, -1)\}$?

6. Let T be the linear transformation from $\Re^2$ to $\Re^2$ which rotates each point by $\pi/2$ radians about the origin in counter-clockwise direction. Write the matrix corresponding to T in the standard basis. How would this matrix change if the basis changes to $\{(1, 1), (1, -1)\}$?