COL 7004

Problem Set

1. The directed Hamiltonian Cycle Problem is as follows: given a directed graph G, is there a cycle which contains all the vertices ? Suppose you have a polynomial time algorithm for this problem. Show that you can also find such a cycle (if it exists) in polynomial time.

2. The undirected Hamiltonian Cycle Problem can be defined similarly as above. The undirected Hamiltonian Path problem is as follows: given an undirected graph G, is there a path which contains all the vertices ? Show that the undirected Hamiltonian path is polynomial time reducible to the undirected Hamiltonian Cycle problem.

3. Show that the undirected Hamiltonian cycle problem is reducible to the directed Hamiltonian cycle problem. Show that the directed Hamiltonian cycle problem is reducible to the undirected Hamiltonian cycle problem.

4. Suppose there is an efficient algorithm for solving the 3-SAT problem. Show how you can also efficiently find a satisfying assignment for a 3-SAT formula which is satisfiable.

5. The set cover problem is as follows: you given a set U, a positive integer k, and a collection $\mathcal{S} = \{S_1, \ldots, S_m\}$ of subsets U. The goal is to check if there is a collection of k subsets in $\mathcal{S}$ whose union is U. Show that the vertex cover problem is polynomial time reducible to the set cover problem.

6. You have a set of friends F whom you're considering to invite, and you're aware of a set of k project groups, $S_1, \ldots, S_k$, among these friends (these sets need not be disjoint). The problem is to decide if there is a set of n of your friends whom you could invite so that not all members of any one group are invited. Prove that this problem is NP-complete. (You can use the fact that the Independent Set problem is NP complete).

7. Given an undirected graph $G = (V, E)$, a feedback set is a set $X \subseteq V$ with the property that $G - X$ has no cycles. The undirected feedback set problem asks: given G and k, does G contain a feedback set of size at most k? Prove that the undirected feedback set problem is NP-complete. (You can use the fact that the vertex cover problem is NP complete).