COL 754

Homework I

Due on 14th August, 2018

1. [**WS**] In the edge-disjoint paths problem in directed graphs, we are given as input a directed graph $G = (V, A)$ and k source-sink pairs $s_i, t_i \in V$. The goal of the problem is to find edge-disjoint paths so that as many source-sink pairs as possible have a path from s_i to t_i. More formally, let $S \subseteq \{1, \ldots, k\}$. We want to find S and paths P_i between s_i and t_i for all $i \in S$ such that $|S|$ is as large as possible and for any $i, j \in S, i \neq j$, P_i and P_j are edge-disjoint. Consider the following greedy algorithm for the problem. Let ℓ be the maximum of $\sqrt{m}$ and the diameter of the graph (where $m = |A|$ is the number of input arcs). For each i from 1 to k, we check to see if there exists an $s_i - t_i$ path of length at most ℓ in the graph. If there is such a path P_i, we add i to S and remove the arcs of P_i from the graph. Show that this greedy algorithm is an $\Omega(1/\ell)$-approximation algorithm for the edge-disjoint problem in directed graphs.

2. [**WS**] Show that for any input to the problem of minimizing the makespan on identical parallel machines for which the processing requirement of each job is more than one-third the optimal makespan, the longest processing time rule computes an optimal schedule.

3. Given an undirected graph $G = (V, E)$, where edges have costs, we want to partition V into two disjoint parts V_1, V_2 such that the total cost of the edges with one end-point in V_1 and the other in V_2 is maximized. Give a greedy 2-approximation algorithm for this problem. Generalize this result to the following problem: we want to partition V into k disjoint parts $V_1, \ldots, V_k$ such that the total cost of the edges whose end-points lie in different parts is maximized. Give a $(1 - 1/k)$-approximation algorithm for this problem.

4. [**V**] Give a greedy algorithm that achieves an approximation guarantee of $\ln n$ for set multi-cover, which is a generalization of set cover in which an integral coverage requirement is also specified for each element and sets can be picked multiple numbers of times to satisfy all coverage requirements. Assume that the cost of picking α copies of a set S_i is $\alpha c(S_i)$.

5. [**V**] In the directed Steiner tree problem, we are given a directed graph $G = (V, E)$. with nonnegative edge costs. The vertex set V is partitioned into two sets, required and Steiner. One of the required vertices, r, is special. The problem is to find a minimum cost tree in G rooted into r that contains all the required vertices and any subset of the Steiner vertices (note that all edges in the tree should be directed away from r). Prove that an $O(\log n)$-approximation for the directed Steiner tree problem implies an $O(\log n)$-approximation for the set cover problem. (Hint: Construct a three layer graph: layer 1 contains a required vertex corresponding to each element, layer 2 contains a Steiner vertex corresponding to each set, and layer 3 contains r.)