Assignment 2, CSL838

Due date: March 27, 2015, 5pm (Fri)

1. Analytically (not numerically in matlab) calculate the Fourier Transform of the following signal. (Hint: you can use the result discussed in class where convolution in the time domain is equivalent to multiplication in the Fourier domain. Also note that addition of two signals in the time domain leads is equivalent to addition of Fourier transforms of the two signals.) (3 marks)

$$s(t) = \begin{cases} |t| & \text{if } -1 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

2. State the Central Limit Theorem (CLT) and the Law of Large Numbers (LLN) (look up any standard source such as Papoulis). (2 marks)

3. (marks: 2) A wireless system uses an 8-PSK modulation scheme and a carrier frequency f_c. Draw the constellation diagram for this modulation scheme. Indicate which signals the two axes correspond to. Assign data bits to the various constellation points such that we minimize the probability of bit error in case a constellation point is incorrectly detected. Assume that all constellation points are transmitted with equal probability. Write the input data bits corresponding to each constellation point in your diagram.

4. (marks: 9) We will study the problem of Peak-to-Average Power Ratio (PAPR) in OFDM signals in this assignment. Given N sub-channels, with corresponding constellation points c_n ($n = 0, \ldots, N-1$), an OFDM baseband signal over one symbol period T_s is:

$$s(t) = \sum_n c_n e^{j2\pi nt/T_s}; \quad 0 \leq t \leq T_s.$$

In practice, we add a cyclic prefix to the signal, which we will ignore for the time being.

In case the phase of the various complex exponentials line up at some point in time, the signal peaks at that instant. This power of the signal $|s(t)|^2$ at that time instant is much larger than the average power of the signal over the entire symbol period. The PAPR is defined as:

$$\text{PAPR} = \frac{\max_{t=0}^{T_s} |s(t)|^2}{\frac{1}{T_s}\int_{t=0}^{T_s} |s(t)|^2 dt}$$

An RF amplifier can amplify input signals by the correct amount provided their power is within some limit. The peaks in the OFDM signal tend to exceed these limits, thus making proper transmission and reception of OFDM signals difficult.

We will determine the PAPR for an OFDM communication system with the following properties.

(a) Data is spread over $N = 16$ channels, and symbol duration $T_s = 32 \times 10^{-8}$s.

(b) The first and last channels, that is those with center frequencies n/T_s for $n = 0$ and $n = N-1$, employ QAM-16 and the rest use BPSK. All channels transmit the same amount of energy per symbol period *on average*.

(c) Given 22 data bits as input for each OFDM symbol (4 bits for each QAM-16 symbol and 1 bit for each BPSK symbol), the complex QPSK/BPSK constellation points, c_n, are generated for $n = 0, \ldots, N-1$. Each constellation point is a complex number whose real part gives the amplitude of the cosine (In-phase part) and imaginary part gives -1 times the amplitude of the sine (Quadrature part). Samples of the baseband signal are then generated from the IFFT of the c_n constellation points:

$$s_k = \sum_{n=0}^{N-1} c_n e^{j2\pi kn/N}; \ k = 0, 1, \ldots, N-1$$

For each possible combination of 22 input bits, compute the PAPR approximately using

$$\text{PAPR} = \frac{\max_k |s(k)|^2}{\frac{1}{N}\sum_k |s(k)|^2}.$$

Plot the following: (I) A histogram of the obtained PAPR values, (II) plot of $|s_k|$ vs. k corresponding to the largest PAPR value obtained, (III) plot of $|s_k|$ vs. k corresponding to the smallest PAPR value obtained. In the titles of plots II) and III), you should give any one sequence of 22 input bits which give rise to these signals.

You may use complex exponentials throughout, for example `exp(j*2*pi*k*n/N)` is a valid expression in matlab. You may also use standard functions available in matlab such as `fft, ifft,hist`.

Submit the following.

- A hardcopy of your code that is commented well. You must not borrow code from any other student or lift code off the Internet.
- Hardcopy of plots mentioned above, and comments on the observations.