Spread-based Heuristic for Sensor Selection in Sensor Networks

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ABSTRACT

In this paper we propose a method for selecting an appropriate subset of sensors with a view to minimize estimation error while tracking a target with sensors spread across in a 2-dimensional plane. In particular, we address the problem of "given N sensors, select $n < N$ sensors to improve the given estimate of target location". Only the selected sensors need to measure distance to the target and communicate the same to the central "tracker". This minimizes the bandwidth and energy consumed in measurement and communication while achieving near minimum estimation error. In this paper, we have proposed that the sensors be selected based on three measures viz. (a) collinearity, (b) spread, and (c) proximity to the target. We use least square error estimation technique to compute the target location using distance measurements subject to multiplicative errors from multiple sensors.

***Index Terms*— Target tracking, Target localization, Sensor selection**

I. Introduction

We consider the problem of estimating the location of a moving target 'T' in a 2-dimensional plane. The target is moving at a speed of at most s kmph. Similarly, the direction of its movement can change at a rate no more than η radians/sec. For a sample trajectory of a moving target in a 2-dimensional plane, please see Figure 1.

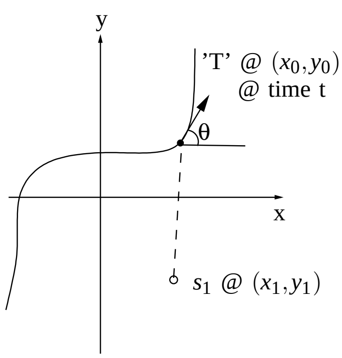

Fig. 1. Trajectory of the target 'T' in 2-dimensional plane.

For the present, we assume that the target is not aware of its own location, or, if it is aware of its location, then it does not share this information with any other device. In either case, we assume that it is possible for sensors, such as s_1 located at $[x_1, y_1]$, to "measure" the distance from/to the target (located at say (x_0, y_0)), and thereby estimate the location of target 'T'. Several methods for measuring distance between a sensor and the target are available. See [1], [2] for methods based on radio signal strength (RSSI) and [3], [4] for methods based on time difference of arrival (TDOA).

Irrespective of the method used to measure distance, the fact remains that all such measurements are subject to error. Two models have been studied in the literature, viz. (a) additive, and (b) multiplicative. These are subsequently described in some detail (see also our earlier paper [5]). In this paper, we largely confine ourselves to using the multiplicative model for errors.

Even though the distance between a given sensor and the target is known only with some error, it is possible to use distance measurements from 3 or more sensors to 'estimate' the location of the target. This, of course, assumes that (a) the location of the sensors is known to the central device responsible for estimating the location of the target, (b) the sensors are time synchronized so that all available sensors can "measure" distance at about the same time, and (c) the sensors are able to communicate their measurements to this central device. In this paper, this central device is referred to as the "tracker".

Since the target is moving and since a sensor must be within a certain distance from the target (before it can detect the presence of the target and measure distance), we assume that there are several sensors spread across the 2-dimensional plane. In fact, we require that an adequate number of sensors be located in and around any given point in the 2-dimensional plane. If ρ number of sensors are randomly and uniformly distributed per unit area then the expected number of sensors within range r_0 of the target at any location, $E(\zeta) = \rho\pi r_0^2$. However, this is not enough. We insist that the probability that the number of sensors capable of measuring distance to the target, ζ, is 3 or more, viz. $P(\zeta \geq 3)$, is nearly 1 (see also [6]). Preferably, there are more than 3 sensors so that one may either (a) use measurements from all available sensors to estimate the location of the target, or (b) compute an estimate based on measurements from a subset of 3 or more sensors suitably selected to minimize estimation error. In this paper we follow the second approach since it allows one to minimize communication overheads and conserve battery power available to sensors. Accordingly, this paper is about suitably selecting n sensors (typically 3 or 4) from a given set of N sensors so as to minimize error in location estimation.

II. Mobile Target Tracking

With this background, we are now in a position to outline the overall scheme for tracking the target as it moves in the 2-dimensional plane.

Let $L_k = [x_k, y_k]$ be the actual location of the target at time t_k. Let $\hat{L}_{k-1} = [\hat{x}_{k-1}, \hat{y}_{k-1}]$ and $\hat{L}_k = [\hat{x}_k, \hat{y}_k]$ be the *estimated* location of the target at time t_{k-1} and t_k, respectively. (Please see Figure 2.) The latter estimate $\hat{L}_k$ is obtained based on (a) an *a-priori* estimate of the target's location $\overline{L}_k = [\overline{x}_k, \overline{y}_k]$, and (b) measurements made at t_k by sensors σ_k^1, σ_k^2, and σ_k^3 located at λ_k^1, λ_k^2, and λ_k^3, respectively. The *a-priori* estimate $\overline{L}_k = [\overline{x}_k, \overline{y}_k]$ may have been obtained based on its estimated location at two time instants t_{k-1} and t_{k-2} (see Figure 2).

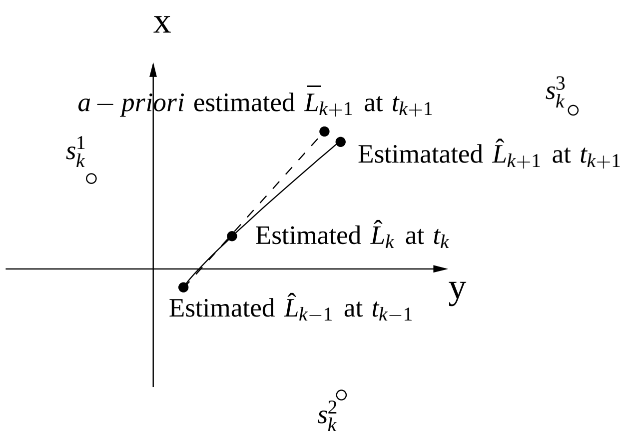

Fig. 2. Estimated location of the target on the basis of *a-priori* estimate.

The new estimate of the location of the target at time t_{k+1} is obtained thus:

- *Step 1:* Given the estimated location of the target $\hat{L}_k$ and $\hat{L}_{k-1}$ at times t_k, and t_{k-1}, respectively, compute an *a-priori* estimate $\overline{L}_{k+1}$ at time t_{k+1} as

$$\overline{L}_{k+1} = \alpha_{k+1}(\hat{L}_k - \hat{L}_{k-1})$$

  where, $\alpha_{k+1} = \frac{t_{k+1}-t_k}{t_k-t_{k-1}}$. (See Figure 2.) This *a-priori* estimate is simply an extrapolation of its location assuming that the average velocity during $[t_k, t_{k+1}]$ is the same as the average velocity during and $[t_k - 1, t_k]$.
- *Step 2:* Given that the target is approximately located at $\overline{L}_{k+1}$, identify an appropriate subset of 3 sensors[1], viz. $\sigma_{k+1}^1, \sigma_{k+1}^2$, and σ_{k+1}^3 from a given subset of sensors $\{s^i\}$.
- *Step 3:* Sensors $\sigma_{k+1}^1, \sigma_{k+1}^2$, and σ_{k+1}^3 obtain distance measurements d_{k+1}^1, d_{k+1}^2, and d_{k+1}^3, respectively, and communicate the same to the central "tracker".
- *Step 4:* The "tracker" computes a least square error estimate of the target location, $\hat{L}_{k+1}$, such that $\sum_{i=1}^{i=3} e_i^2$ is minimized. Here, $e_i = \|\hat{L}_{k+1} - \lambda_{k+1}^i\|^{\frac{1}{2}} - d_{k+1}^i$, and $\|\hat{L}_{k+1} - \lambda_{k+1}^i\|$ is the Euclidean distance between estimated location of target $\hat{L}_{k+1}$ and $\lambda_{k+1}^i = [x_{k+1}^i, y_{k+1}^i]$, the location of sensor σ_{k+1}^i. This method is described in Section IV.

[1]or possibly more.

In this paper, we focus attention on Step 2, viz. that of selecting an appropriate subset of 3 (or more) sensors with a view to minimize the estimation error. Note that only the selected sensors need to measure distance to the target and communicate the same to the central "tracker". This minimizes the bandwidth and energy consumed in measurement and communication while achieving near minimum estimation error.

Section III describes related work in the area of target tracking and sensor selection. Location estimation has been described in Section IV, while Section V and Section VII describe the method and the algorithm for sensor selection, respectively. We present simulation results in Section VII.

III. Related Work

Several researchers have focused attention on the problem of estimating the location of a fixed target, given measurements from a subset of sensors. Their findings are reported in [1], [3], [7], [8], [9], [10], [11], [12], and [13]. Their approaches differ from each other on the basis of (a) the number of sensors required, (b) the nature of measurements, and (c) the technique used to estimate the location.

- Priyantha et al [3] estimate location of the target using trilateration using distance measurements based on TDOA from 3 different sensors.
- Bahl et al [1] estimate location of the target using trilateration, but using RSSI measurements. They build a radio map of the site and locate targets based on its radio signal strength measurements.
- Triangulation is used in robotics [13] to estimate the location of the robot. This requires 3 or more angle of arrival (AoA) measurements to estimate the location of the robot.
- Information theoretic approach has been used in sensor networks in [8]. Bayesian filtering is used to estimate location of the target. Multiple measurements of different types can be used in this approach.

In this paper, however, we use distance measurements from a given set of 3 or more sensors. The estimate is a least square error estimate.

Tracking of mobile targets using sensor networks has been studied in [14], [15], [16], [17], and [18].

- In [14], [15] the location of the target is approximated by the location of a sensor when the target comes within the range of that sensor. The resulting resolution is the same as the range of sensors. The resolution can be improved if measurements from multiple sensors are considered.
- In [17] the location of the target is computed by minimizing a function of the error in making acoustic measurements. Groups of sensors called clusters are formed. Acoustic measurements from all the sensors in the cluster

are used by the cluster head to compute the location of the target.

- In [16], [18], the target is tracked by a group of sensors that form a logical tree for the purpose of sharing measurement data.

In this paper, and as described earlier in Section II, a group of sensors in and around an *a-priori* estimated location of the target is selected. This group changes as and how the target moves within the 2-dimensional plane.

With respect to sensor selection, several papers [5], [8], [9], [11], and [12] have used different approaches.

- Zhao et al have proposed in [8] and [9] that sensors responsible for measuring distance (using acoustic signals) be selected such that (a) the use of communication bandwidth is minimized, and (b) the error in locating the target is minimized. The selection procedure however, assumes that an *a-priori* estimate of the location is available. Further, the estimate is based on Bayesian maximum likelihood estimator.
- Wang et al [11] also assume *a-priori* knowledge of the target location also expressed as a Gaussian probability distribution function. The error in TDOA or AoA measurements by sensors is also specified as a Gaussian probability distribution. While the estimate of the target location is based upon Bayesian filtering, the sensors selected to provide measurements are those that maximize the entropy difference between the *a-priori* and *posteriori* estimates of the target location.
- Isler et al [12] use AoA measurements from multiple sensors. The target location is estimated based on the region of intersection of two or more 2-dimensional cones resulting from uncertainty in AoA measurements. Sensors which minimize the area of such intersection are selected. This scheme requires an *a-priori* knowledge of the location of the target.
- In our earlier work [5], we have considered the problem of selecting the n^{th} sensor given (a) distance measurements from $(n-1)$ sensors (these measurements are subject to multiplicative errors), and (b) an *a-priori* estimate of the target location. The n^{th} sensor thus selected minimizes the area of intersection of error annulus.

The work reported in this paper is different from our earlier work. Here, we have proposed that the sensors be selected based on three measures viz. (a) collinearity, (b) spread, and (c) proximity of sensors from the target. We use least square error estimation technique to estimate the target location.

IV. Target Location Estimation

Before discussing ways to select a subset of sensors, we discuss the method used to obtain a least square error estimate of the location of the target at time t_k. For convenience we drop the subscript k in t_k and instead discuss the estimate at time t. The estimation problem can be stated thus:

Given distance measurements d^1, d^2, $\ldots$, d^n from sensors $s^1, s^2, \ldots, s^n$, respectively located at $\lambda^1 = [x^1, y^1]$, $\lambda^2 = [x^2, y^2]$, $\ldots$, $\lambda^n = [x^n, y^n]$, compute an estimate $\hat{L} = [\hat{x}, \hat{y}]$ such that

$$\sum_{i=1}^{i=n} \{ \sqrt{(\hat{x}-x^i)^2 + (\hat{y}-y^i)^2} - d^i \}^2$$

is minimum. The measurements are possibly subjected to unknown measurement errors.

This is a standard non-linear optimization problem. Of the standard algorithms available, we have experimented with (a) Steepest Descent algorithm [19], and with (b) Levenberg Marquardt algorithm [19] to compute the optimal $[\hat{x}, \hat{y}]$. In either case, the method requires an initial "guess". We have proposed that this guess be obtained by solving $\binom{n}{2}$ linear equations[2]. If $n > 3$, then the system of $\binom{n}{2}$ linear equations is overdetermined, and therefore the initial guess is itself based on a linear least square error solution to these equations.

Our experience shows that (a) the number of iterations in Steepest Descent method were less than 20, while these were less than 6 in case of Levenberg Marquardt, and (b) the above method for computing the initial guess is reasonably adequate in helping one to descend to the optimum.

No. of Sensors	**Error in Optimized Estimate**
3	7 m
4	2.6 m
5	2.5 m

TABLE I

Location estimation accuracy and number of measurements.

The third and somewhat more important outcome of the experiments conducted by us helps to demonstrate that while 3 sensors is an absolute must to locate the target, availability of distance measurement from a 4^{th} sensor significantly reduced the estimation error. However, a measurement from a 5^{th} sensor provides only marginal improvement in the estimate. See Table I.

V. Sensor Selection

The positions of sensors relative to each other and relative to the position of the target play an important role in location estimation accuracy. The sensor selection technique proposed in this paper is based on three factors, each of which affects the accuracy of estimated location of the target. These are (a) collinearity of sensors, (b) deviation from the ideal spread, and (c) proximity of selected sensors from the target.

A. Collinearity of Sensors

Consider the distribution of sensors s^1, s^2, and s^3 in a 2-dimensional plane shown in Figure 3, and let the distance measurement be d^1, d^2, and d^3. If, for the moment, we assume that the error in distance measurements in near zero, then it can be concluded that the target is either located at position A, or at position B. Note, this conclusion could also have been

[2] These are obtained by subtracting equations of the type $\sqrt{(\hat{x}-x^i)^2+(\hat{y}-y^i)^2} - d^i = 0$ from one another, thereby resulting in $\binom{n}{2}$ linear equations.

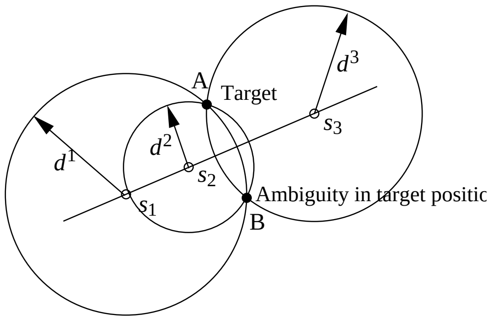

Fig. 3. Ambiguity in target location in case of collinear sensors.

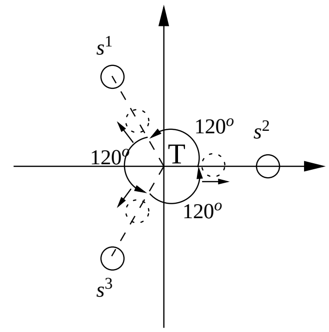

Fig. 4. Role of proximity of sensors from the target.

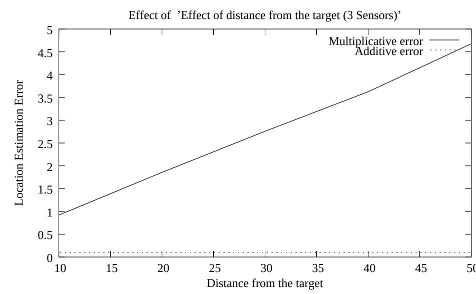

Fig. 5. Distance δ from the target vs. Error in estimated location.

arrived at using distance measurements from sensors s^1 and s^2, or s^1 and s^3, or s^2 and s^3. That is, measurement from a third sensor does not add value. The reason is, of course, that the sensors s^1, s^2 and s^3 are collinear. Distance measurement from another sensor which is not collinear will additionally be required to resolve whether the target is at location A, or at B. We, therefore, define a measure of collinearity, that will subsequently be used to select an appropriate subset of sensors. This measure of collinearity is defined as the residual error resulting from a linear least square fit through the given n sensors. That is, the collinearity coefficient

$$\Phi = min_{(m,c)} \sum_{i=1}^{n} \{y^i - mx^i - c\}^2 \tag{1}$$

where m is the slope and c is the y-intercept of a straight line fit $y = mx + c$ through s^1, s^2, and s^3.

Note, if the collinearity coefficient is small, the sensors are almost collinear. In that case the location estimation error is possibly large[3]. But if the collinearity coefficient is large, the location estimation error is likely to be small. These results have been discussed at length in our earlier work [5].

B. Proximity of sensors to target

Having established one heuristic based on collinearity of sensors for selecting a subset of sensors, we now discuss how far the sensors are preferably placed. Below, we establish the fact that given a choice, the sensors are preferably placed as close as possible to the target. This conclusion is based on the assumption that measurement error is multiplicative in nature. That is, if the actual distance between a sensor s^i and the target is δ^i, and the measured distance is d^i, then the error is assumed to be $| e^i | = | d^i - \delta^i | = \varepsilon\delta^i$, where the parameter ε, $0 \leq \varepsilon \leq 1$, helps indicate the amount of error as a percentage of the actual distance. As an example, if the measurement error is 20% ($\varepsilon = 0.2$), and the actual distance is 20m, then the measured distance is between 16m to 24m.

We will now show how the uncertainty in the location of the target increases with increase in the distance of the sensor from the target.

[3]Another reason for the large error in estimated location is that we can not have a good initial "guess" of the location of the target.

Consider a case where three sensors s^1, s^2, and s^3 are at distance δ from the target with equal visual angles[4], $\alpha^i = \frac{2\pi}{3}$ (see Figure 4). Consider the region of intersection of the error annuli corresponding to the computed distance between the *a-priori* estimated location of the target and the three sensors. Clearly, the region formed around the *a-priori* estimate of its location is the one in which the probability that the target is located is the maximum. For simplicity, this latter region is approximated by a hexagon obtained by intersecting bands formed by tangents to the error annuli. The largest diagonal of the parallelogram corresponds to the *maximum* uncertainty in the location of the target. The length of largest diagonal,

$$l = \frac{\varepsilon\delta}{sin(\frac{\pi}{3})}$$

where ε indicates the amount of error in distance measurement as a percentage of the actual distance. The error in location estimation is linearly proportional to the distance of the sensors from the target. This is also confirmed with the help of simulations where the δ, the distance of the three sensors s^1, s^2, and s^3 (see Figure 4) is simultaneously increased from 10m to 50m and the error in estimated location is observed. This is plotted in Figure 5.

We are now ready to consider the selection of (say) 3 sensors based on their proximity from the target. Let the target 'T' be actually located at $[0,0]$. Let sensors s^1, s^2, s^3, and s^4 be located at locations λ^1, λ^2, λ^3, and λ^4, respectively. Then, given that everything else is the same, e.g. the extent of

[4]The visual angle is the angle incident upon the target from the two sensors.

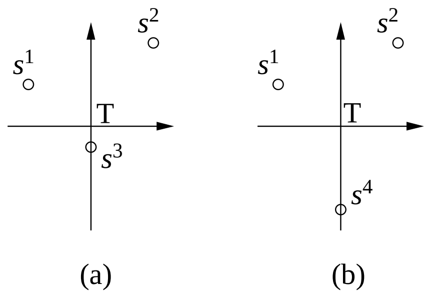

Fig. 6. Selection of sensor based on proximity to the target.

collinearity and spread (discussed in Subsection V-C), which of the following two selections of sensors (a) s^1, s^2, and s^3 (see Figure 6 (a)), or (b) s^1, s^2, and s^4 (see Figure 6 (b)) is likely to result in a better estimate of the location of the target.

The answer to this question is clearly (a) since we assume multiplicative measurement error because of which $\mid e^3 \mid = \varepsilon \mid \delta^3 \mid < \mid e^4 \mid = \varepsilon \mid \delta^4 \mid$. As a consequence, the estimation error is likely to be smaller in case of (a) as opposed to case (b).

But in order to address a similar question in respect of two selections of (a) sensors s^1, s^2, and s^3, and (b) sensors s^4, s^5, and s^6, where none of the sensors is common, we define a cumulative measure for proximity $\grave{E} = \sqrt{\sum_{i=1}^{n} \mid e^i \mid^2} = \varepsilon\sqrt{\sum_{i=1}^{n} {\delta^i}^2}$.

But note, the above formula for $\grave{E}$ refers to the actual distance δ^i, which, however, are not available. For that matter, even measured distance d^i is unavailable since measurements are made only *after* a subset of sensors are suitably selected. Necessarily, then, the measure of proximity must be based upon the estimated distance between the given sensor s^i and an *a-priori* estimate of the location of the target. Thus if an *a-priori* estimate of the target is $\bar{L}$, and the sensors (under consideration) are s^1, s^2, $\ldots$, and s^n, then the proposed measure of the proximity is

$$E = \varepsilon\sqrt{\sum_{i=1}^{n} \|\lambda^i - \bar{L}\|^2} \qquad (2)$$

Clearly, if one were to select the n sensors for which $e^i = \varepsilon\|\lambda^i - \bar{L}\|$, $i = 1,2,\ldots,n$ is the smallest then the above measure of proximity is minimized. However, this approach is not going to be useful since one would need to consider other measurements, such as collinearity and spread (discussed in next Subsection) before selecting a subset of sensors.

C. Spread

If the sensors responsible for distance measurements are well distributed around the target, for instance with equal visual angles between each other, then the estimation error is likely to be small. This is so since the area of intersection of the error annulus will be small [20]. (Please see Figure 7 (a) for ideal distribution of 3 sensors and Figure 7 (b) for non-ideal distribution of 3 sensors.)

We therefore propose a measure of the extent to which the visual angles deviate from the ideal spread of sensors. The

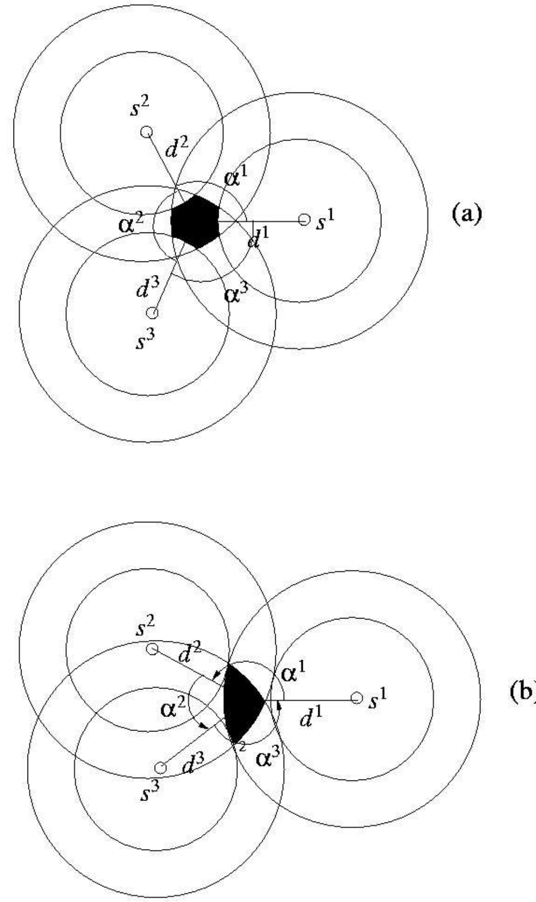

Fig. 7. Region of intersection of annulus for 3 sensors.

ideal situation corresponds to n visual angles $\alpha_i = \frac{2\pi}{n}$, for $i = 1,2,\ldots,n$. Please see Figure 7 (a) for an example of the ideal spread for 3 sensors where $\alpha_i = \frac{2\pi}{3}$ for $i = 1,2,\ldots,3$. A measure which captures deviation from the ideal spread in case of n sensors, viz. $\alpha_i = \frac{2\pi}{n}$, $i = 1,2,\ldots,n$, can be defined thus:

$$\Delta(\lambda^1,\lambda^2,\ldots,\lambda^n) = \sqrt{\sum_{i=1}^{i=n}\left(\frac{2\pi}{n} - \alpha_i\right)^2} \qquad (3)$$

where λ^i, $i = 1,2,\ldots,n$ is the location of sensor[5] s^i, α_i, $i = 1,2,\ldots,n$ are the actual visual angles between sensors s^i and s^{i+1}, respectively. In the ideal case when $\alpha_i = \frac{2\pi}{n}$, the deviation $\Delta(\lambda^1,\lambda^2,\ldots,\lambda^n) = 0$. When all the sensors are collinear with the target and are on one side of the target, this measure of the spread is the maximum. In this case, $\alpha_i = 0$, i = 1, 2, …, n-1, and $\alpha_n = 2\pi$. As a consequence, $\Delta(\lambda^1,\lambda^2,\ldots,\lambda^n) = 2\pi\sqrt{\frac{n-1}{n}}$, which for $n = 3$, is 5.13, and for n = 4, Δ = 5.44.

Consider a case where three sensors s^1, s^2, and s^3 are placed as shown in Figure 8. Sensors s^2 and s^3 are placed at an angle θ with sensor s^1. An increase in θ from 0 to π results

[5] These are suitably sorted (and thereafter numbered) according to the angle that it forms with the x-axis (assuming that the target is at $[0,0]$).

in different values of deviation from the ideal spread, Δ, as shown in Figure 9.

Now consider the region of intersection of the error annuli corresponding to the computed distance between the *a-priori* estimated location of the target and the three sensors (see Figure 7). The latter region is approximated by a polygon obtained by intersecting bands formed by tangents to the error annuli. The largest diagonal of the parallelogram corresponds to the *maximum* uncertainty in the location of the target. The length of the largest diagonal, l, and the corresponding error in estimated location (using least square error technique) vs. θ are shown in Figure 10. From Figures 9, 10, it can be observed that (a) the error in estimated location increases with an increase in the length of the largest diagonal, l, and (b) with an increase in the value of deviation from the ideal spread, Δ, the error in estimated location increases too.

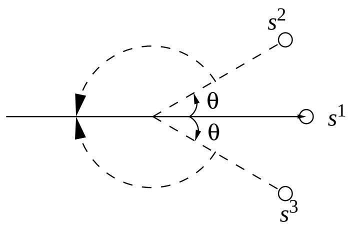

Fig. 8. Role of deviation from the ideal spread, Δ.

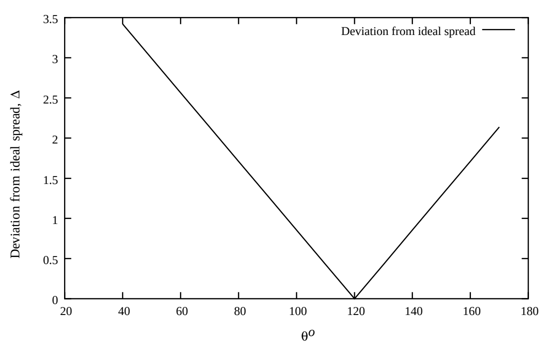

Fig. 9. Deviation from the ideal spread Δ vs. θ.

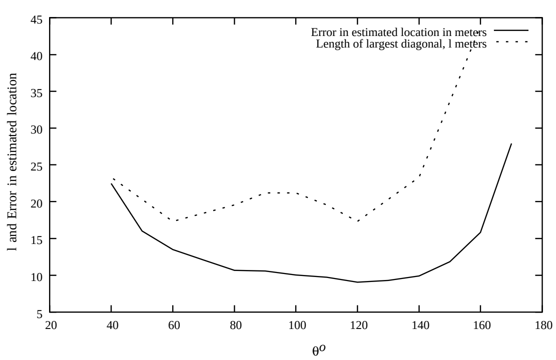

Fig. 10. Length of larger diagonal, l, and Error in estimated location vs. θ.

In a more generalized scenario, to see the effect of deviation from the ideal spread on the error in estimation of location of the target, we placed n sensors, all at the same distance from the target. This ensured that the effect of error in distance measurement was constant. The n sensors (3 or 4 as the case may be) were systematically placed at a wide variety of locations, thereby yielding different values for deviation from the ideal spread.

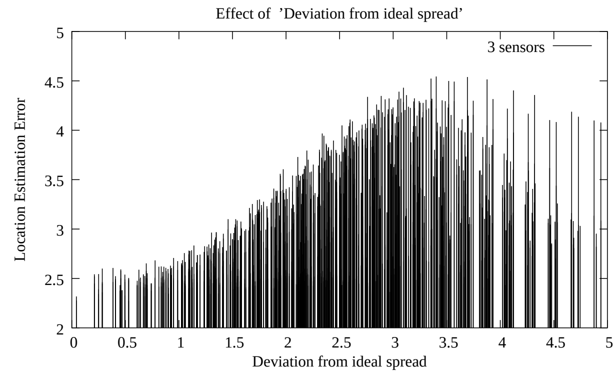

Fig. 11. Effect of spread of 3 sensors on location estimation error.

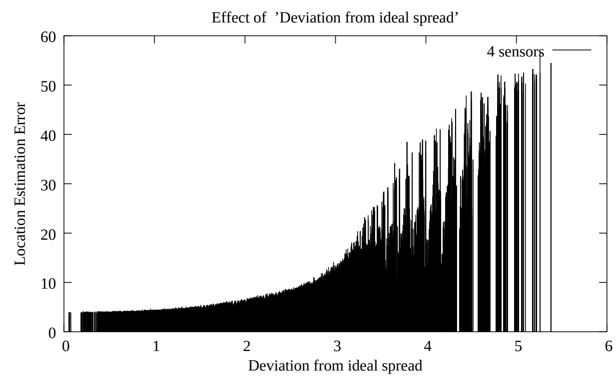

Fig. 12. Effect of spread of 4 sensors on location estimation error.

Figure 11 shows the results with 3 sensors. The location estimation error is low for placements that result in smaller deviation from the ideal spread, Δ. Similar behavior has been observed in case of 4 sensors (see Figure 12).

Here again, as with measure of proximity, the deviation for the ideal spread must be calculated on the basis of an *a-priori* estimate of the location of the target.

VI. "CSP" ALGORITHM FOR SENSOR SELECTION

In this paper, we propose that a subset of n sensors, from those sensors which have adequate available battery power, be selected such that

- the collinearity coefficient is maximized,
- deviation from the ideal spread is minimized, and
- the measure of proximity to the target is minimized.

This is a multi-objective optimization problem. Below, we propose an algorithm, "CSP" to solve this problem. Though it results in an approximate solution, the resultant error in estimated location is generally expected to be near minimum. For a given ϕ_0, and δ_0:

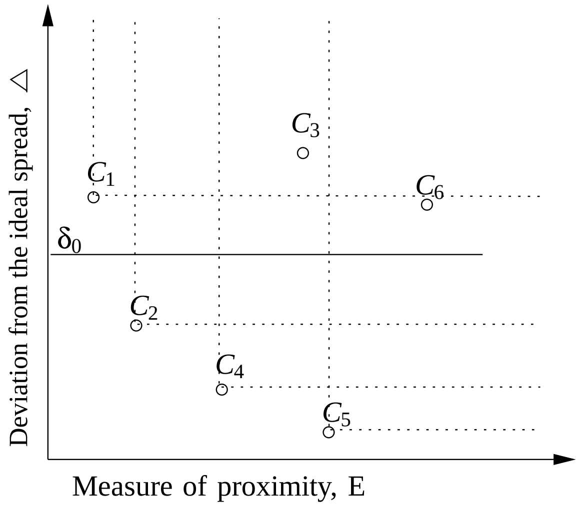

Fig. 13. Plot of selections on the basis of their proximity to the target and deviation from the ideal spread.

- *Step 1:* Eliminate a selection of subset of n sensors (out of $\binom{N}{n}$) for which the collinearity coefficient, $\Phi \leq \phi_0$.
- *Step 2:* Of the remaining selections, consider only those for which deviation from the ideal spread, $\Delta < \delta_0$.
- *Step 3:* Finally, select that subset of n sensors for which E is the minimum.

Having eliminated all selections that are near-collinear, we are left with subsets of sensors (or selections) that need to be compared on the basis of their respective deviations from the ideal spread, Δ, and measure of proximity, E. This comparison is best carried out by plotting them on a 2-dimensional plot, as in Figure 13.

If one were to compare the 6 different selections C_1 through C_6, it would be clear that C_3 and C_6 are ”poorer” selections compared to C_2 or C_4 in both respects, Δ, and E. However, the same is not true of C_1, C_2, C_4, C_5. They are ”equally good”. They present different trade-off between the two criteria for selecting sensors.

An application of Step 2 of the above algorithm, however, suggests that we discard C_1 on the basis of ”deviation from the ideal spread” being large. Step 3 then identifies C_2 to be the ”best” selection.

The complexity of the above algorithm is $O(nN^3)$ where N is the total number of sensors which have detected the target, n is the number of sensors to be selected, and $n << N$. However, this should not be a major deterrent since we propose to replace one sensor every time a new estimate is required to be obtained. Specifically, we assume that as the target moves, if sensors $\{s^1, s^2, s^3\}$ have made measurements at time t_k, then at time t_{k+1}, we drop one of the sensors s^1, s^2, or s^3 and select a sensor s^4 suitably so as to minimize the error in estimated location of the target. In this case, the number of possible selections to be analyzed is $3(N-2)$. As a result, the complexity of the algorithm is $O(N)$.

The above algorithm considers the three parameters viz. collinearity, spread, and proximity, in that order. Clearly, there are other possibilities resulting from considering the three parameters in different order. These options are tabulated in Table II and evaluated in detail using simulation.

Algorithm	Code	Order of parameters considered
1	CSP	Collinearity, Spread, Proximity
2	SCP	Spread, Collinearity, Proximity
3	CPS	Collinearity, Proximity, Spread
4	SPC	Spread, Proximity, Collinearity
5	PCS	Proximity, Collinearity, Spread
6	PSC	Proximity, Spread, Collinearity

TABLE II

ALGORITHMS WITH DIFFERENT ORDERING OF THE THREE PARAMETERS.

VII. SIMULATION RESULTS

Simulations were carried out with N sensors (N = 10 or 20) randomly placed within 100m of the ”actual” location of the target. The sensing range is also $r_0 = 100$m. The measurement error was assumed to be 15%, that is $\varepsilon = 0.15$. The target was assumed to be at $[0,0]$ and the *a-priori* location of the target was also assumed to be $[0,0]$. n = 3 sensors were subsequently selected according to the algorithm given above. Therein, the threshold for collinearity coefficient, $\phi_0 = 40m^2$, and the threshold for the deviation from the ideal spread, δ_0 = 2 or δ_0 = 4. The results averaged over ten random placements of N sensors have been tabulated in Table III and IV.

We have also compared the results obtained using the above algorithm with other methods for sensor selection that are based on (a) minimum measure of proximity to the target *only*, and (b) minimum deviation from the ideal spread *only*. The error in estimated location based on our algorithm is also compared with the error resulting from selecting the ”best” possible subset of sensors. The latter is obtained by exhaustively computing the least square error estimate for each possible subset of sensors. From Table III it may be observed that the proposed algorithm results in significantly lower estimation error when compared with algorithms based on (a) proximity only, and (b) deviation from the ideal spread only. In fact, the resulting estimation error is close to the error resulting from the ”best” possible subset of sensors.

N	Selection based on proximity only	Selection based on spread only	Selection based on proposed algorithm ($\delta_0 = 2$)	Absolutely The best subset of sensors
10	16.55m	7.66m	6.84m	5.53m
20	8.75m	7.51m	5.02m	4.16m

TABLE III

n = 3, ϕ_0 = 40m^2.

Further, it is observed that the error is significantly lower for $\delta_0 = 2$, as opposed to $\delta_0 = 4$ (see Table IV). This can be explained thus: by assigning larger threshold, viz. $\delta_0 = 4$, one is effectively reducing the significance of the parameter, ”deviation from the ideal spread”. Therefore, it is important

N	Selection based on proposed algorithm ($\delta_0 = 2$)	Selection based on proposed algorithm ($\delta_0 = 4$)	Absolutely The best subset of sensors
10	6.84m	8.50m	5.53m
20	5.02m	6.42m	4.16m

TABLE IV

$n = 3, \phi_0 = 40m^2$.

ϕ_0	δ_0	**CSP (N = 10)**	**Min. poss. (N = 10)**	**(CSP (N = 20)**	**Min. poss. (N = 20)**
50%	50%	6.82m	5.53m	5.44m	4.16m
50%	25%	6.58m	5.53m	5.04m	4.16m
25%	50%	7.72m	5.53m	6.77m	4.16m
25%	25%	7.36m	5.53m	6.32m	4.16m
75%	25%	6.33m	5.53m	4.76m	4.16m
25%	75%	8.15m	5.53m	7.65m	4.16m

TABLE V

"CSP" ALGORITHM WITH DIFFERENT THRESHOLDS OF ϕ_0, AND δ_0, (n = 3).

N	CSP	CPS	SCP	SPC	PCS	PSC	Min. poss.
10	6.82m	7.21m	7.78m	9.60m	7.26m	10.11m	5.53m
20	5.44m	7.29m	6.53m	11.23m	7.24m	11.62m	4.16m

TABLE VI

ALGORITHMS WITH MEDIAN THRESHOLDS (n = 3).

to choose appropriate values of the thresholds for parameters, "deviation from the ideal spread", δ_0, "coefficient of collinearity", ϕ_0, and possibly e_0, where e_0 is the threshold for "measure of proximity". This is a difficult problem. One method of arriving at a suitable value for the threshold is to retain a certain fraction (say 50%) of candidate selections. As a result, the proposed algorithm "CSP", above, may be rewritten as:

- *Step 1a:* Compute ϕ_0 as the median collinearity coefficient, considering all selections of subsets of n sensors (out of $\binom{N}{n}$).
- *Step 1b:* Eliminate all selections of n sensors for which the collinearity coefficient, $\Phi \leq \phi_0$.
- *Step 2a:* Compute δ_0 as the median deviation from the ideal spread, considering only remaining selections.
- *Step 2b:* Of the remaining selections, consider only those for which deviation from the ideal spread, $\Delta < \delta_0$.
- *Step 3:* Finally, select that subset of n sensors for which E is the minimum.

The results corresponding to the above algorithm are tabulated in Table V together with results that correspond to an algorithm where 75%, 50%, and 25% of the possible selections of subset of sensors are retained (and for both N = 10, and N = 20). Two things can be concluded, viz. (a) the parameter, deviation from the ideal spread should be given due weightage, and (b) by suitably identifying the percentage of selections to be retained one can approach the "best possible selection".

Further, we compare results of "CSP" algorithm with results from other algorithms where the three parameters, viz. collinearity, spread, and proximity are considered in different order (see Table II). These results are presented in Table VI with thresholds calculated based on medians. It can be observed that the "CSP" algorithm achieves near minimum estimation error as compared to other algorithms.

For N = 20, the error resulting from the proposed algorithm is always smaller since with larger number of randomly placed nodes there is a possibility of finding a "better" subset of sensors.

VIII. CONCLUSION

We consider the problem of estimating the location of a moving target 'T' in a 2-dimensional plane. We focus attention on selecting an appropriate subset of 3 (or more) sensors with a view to minimize the estimation error. Only the selected sensors need to measure distance to the target and communicate the same to the central "tracker". This minimizes the bandwidth and energy consumed in measurement and communication while achieving near minimum estimation error.

In this paper, we have proposed that the sensors be selected based on three measures viz. (a) collinearity (b) spread, and (c) proximity to the target. We use measurements subject to multiplicative errors from multiple sensors. Further we use least square error estimation technique to compute the target location.

Our experiments have shown that while 3 sensors is an absolute must to locate the target, availability of distance measurement from a 4^{th} sensor significantly reduces the estimation error. However, a measurement from a 5^{th} sensor can only marginally improve upon the estimate. We have presented results of our algorithm with n = 3.

The sensor selection is done by the central "tracker" and only selected sensors measure distance to the target and communicate them to the central "tracker" for estimating the target location.

We propose that a subset of n sensors, from those sensors which have adequate available battery power, be selected such that (a) the collinearity coefficient is maximized, (b) deviation from the ideal spread is minimized, and (c) the measure of proximity to the target is minimized. This is a multi-objective optimization problem. The proposed algorithm gives an approximate solution. The results have shown that it is possible to achieve near minimum error in estimated location of the target.

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