1 Answer 2

We can simulate a k-tape Turing Machine M by a one tape TM N having 2k tracks. Half of the these 2k tracks hold the tapes of M and other half of the tracks each holds a marker that indicates where the head for the corresponding tape of M is currently located. To simulate a move of M, N's head must visit the k head markers. Now N knows the symbols being scanned by the heads of M and it also knows the state of M. So, N knows the move which M is going to make. Then N revisits the head markers and changes the symbols on the tracks and also moves the head markers left or right.

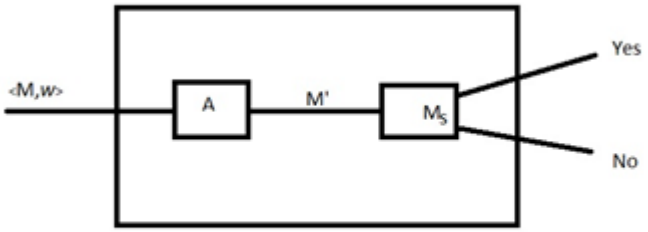
After n moves of M the tape head markers cannot have separated by more than 2n cells. So, if N starts at the leftmost marker it has to move no more than 2n cells rightwards to find all the head markers. It can then move leftwards changing the contents of the simulated tapes of M and moving the head markers left or right. Time required for this is less than 4n+2k. So, to simulate n moves of M, N takes $O(n^2)$ time.

For answer of the second part of the question please refer to section 8.6.3 of the textbook.

2 Answer 7b

The problem is not decidable.

Suppose the problem is decidable. We build a Turing Machine Ms which tells us if for a given machine M there is a w such that M visits all of its states during the computation on w. Now we reduce Lu to Ls.



Algorithm A works as follows:

- It outputs the code of a machine M'.
- IF M accepts w then M' enters a special state q' such that all states of M are visited from q'.

- If M does not accept w then q' is never reached.

So, clearly Ms will output Yes if M accepts w and No if Ms doesn't accept w .

3 Answer 7.c

The problem is decidable.

We can simulate the TM M using a multi-tape TM M'. M' uses one of its tracks to check if the head has reached the cell B. On one track of M' we record all the ID's we have seen so far. As B is bounded, the number of ID's is finite. At every step we check if we have already seen the ID or not. If it is new we record it otherwise the machine halts and outputs NO. If the head ever reaches cell B then we halt and output Yes. Since the number of ID's is finite the machine will eventually halt.