CSL 705 Theory of Computation, Tutorial Sheet 4

1. Show that a $T(n)$ time bounded k tape Turing machine can be simulated by a 1 tape machine in $O(T^2(n))$ steps.
   Use this result to show how a Random Access Machine can be simulated by a 1 tape TM incurring at most a polynomial overhead.

2. Show that any arbitrary boolean function of k variables can be expressed by a CNF formula of atmost 2^k clauses.
   Note: When k is constant this is also constant, and therefore the CNF formula in the proof of Cook-Levin thm expressing the transition function of the NDTM is of bounded size as it only involves 4 cells.

3. Can you design an efficient algorithm that satisfies at least 50 % of the clauses in a 3 CNF Boolean formula. How about 66% ?

4. Consider the family of boolean circuits (consisting of **AND, OR, NOT** gates) of size S. Show that in order for the circuit to compute any of $n-ary$ boolean functions, S should be $\Omega(2^n/\log n)$.
   Note that it shows that there are functions for for which exponential size circuits are necessary but these functions may not be in NP !
   Hint: Model the circuit as a labelled graph on S nodes and use a counting argument on the possible number of circuits.

5. In the proof of Cook-Levin theorem, show that for $x \in L$, and the reduction function f, a certificate y (non-deterministic choices) is mapped to a unique certificate of $f(x)$. Recall that $f(x)$ is a CNF formula and a certificate for CNF formula is a satisfying assignment.

6. Show that if a boolean formula in CNF contains at most one un-negated literal, then the satisfiability problem can be solved in polynomial time.
   Note: Such clauses are called Horn clauses

7. Assuming that 3D matching is NP complete, show that the problem of partition is NP complete.
   Hint: Work out the details of the proof outlined in class.

8. In the partition problem, if the sum of the integers are bounded by a polynomial,show that it is possible to design a polynomial time algorithm.

9. Show that the problem of *independent set* is NP complete.

10. Show that the problem of sub-graph isomorphism problem is NP complete. In this problem, the input are two graphs G_1 and G_2, does G_1 contain a copy of G_2 as a subgraph.