1. Design DFA for the following languages over \{0, 1\}:

(a) The set of all strings such that every block of of five consecutive symbols have at least two 0’s.
(b) The set of all strings beginning with a 1 which interpreted as an integer is congruent to zero modulo 5.
(c) The set of strings with an equal number of 0’s and 1’s such that each prefix has at most one more 0 than 1’s and at most one more 1 than 0’s.
(d) The set of strings not containing the substring 110.

2. Design NFA for the following languages:

(a) The set of strings over \{0, 1\} such that some pair of 0’s is separated by a string of length \(4i \geq 0\).
(b) The set of strings over \{a, b\} that have the same value when multiplied from left to right as from left to right. The rules of multiplication are \(a \times a = b, b \times b = a, a \times b = b, b \times a = b\). Note that \((a \times b) \times b = a\) and \((a \times (b \times b)) = b\), i.e. they are not same, i.e. it is not associative.
(c) The set of strings of the form \(x \cdot w \cdot x^R\) where \(x, w\) are strings over 0,1 of non-zero length.

3. Prove or disprove the following about regular expressions \(r, s, t\) where \(r = s\) implies \(L(r) = L(s)\):

(a) \(r(s + t) = rs + rt\)
(b) \((r^*)^* = r^*\)
(c) \((r^*s^*)^* = (r + s)^*\)
(d) \((r + s)^* = r^* + s^*\)

4. Which of the following are regular sets - Prove them:

(a) \(\{0^{2^n} | n \geq 1\}\)
(b) \(0^m0^n0^{m+n} | m, n \geq 1\}\)
(c) \(\{0^n | n \text{ is prime}\}\)
(d) The set of all strings with equal number of 0’s and 1’s.
(e) Set of all palindromes over 0,1.
(f) \(\{xx^Rw | x, w \in (0 + 1)^+\}\)

5. Let \(L\) be a regular set. Which of the following are regular:

(a) \(\{a_1a_3 \ldots a_{2n-1}a_1a_2a_3 \ldots a_{2n} | a_1a_2a_3 \ldots a_{2n} \text{ is in } L\}\)
(b) \(MAX(L) = \{x \text{ is in } L | \text{ no extension of } x \text{ is in } L\}\)
(c) \(L^R = \{x | x^R \text{ is in } L\}\)
(d) \(\frac{1}{2}(L) = \{x | \text{ for some } y \text{ such that } |x| = |y|, xy \in L\}\).

6. A set of integers is linear if it is of the form \(\{c + pi | i \geq 0\}\). A set is semilinear if it is a finite union of linear sets. Let \(R \subseteq 0^*\) be regular. Prove that \(R\) is semilinear.

7. What is the relationship between class of regular sets and the least class of languages closed under union, intersection and complement containing all finite sets?