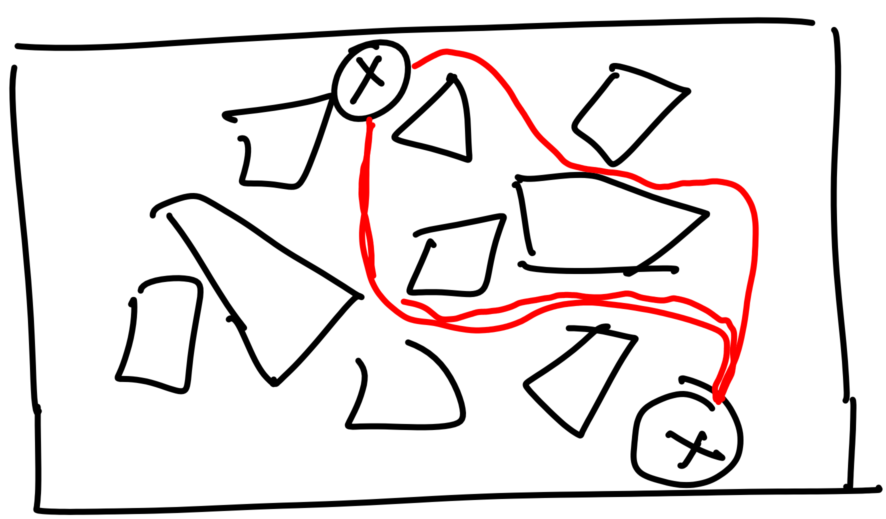
CSL 630   Lecture 26, Nov 10

Can we design an algorithm for any given problem? NO

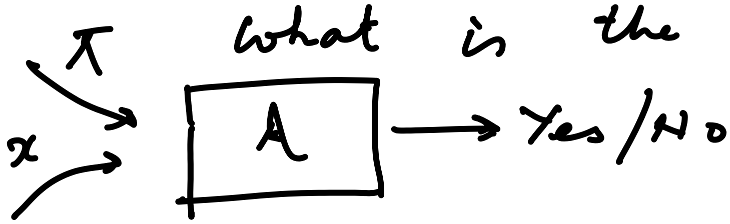
Eg. - Given a number, find the factors (discrete problem)

- Given two points in a room, find a path to move an object between the two points.



- Given a C program π and an input x what is the <u>output</u> of π on x

does it terminate?



There exists "undecidable" problems.

Can we design "efficient" algorithm for those problems for which we can design

$$n^{n^{n^{\cdot^{\cdot}}}} \quad \ldots \quad 2^n$$

polynomial time is considered acceptable

$$n^3 \quad n^4 \quad \ldots \quad n^{1000}$$

Classification of efficient / not efficient

polynomial — not polynomial

We can design a polynomial time algorithm

$2^{\log^2 n}$

$(2^{\log n})^{\log n}$

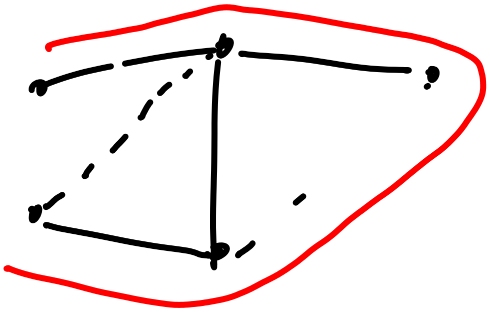
$n^{\log n}$

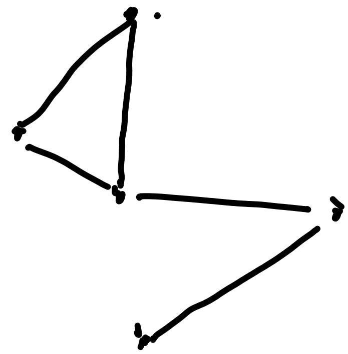
$n^{\log^* n}$

- ~~I cannot design a polytime algo~~
- Polytime algorithm is not known to exist
- No one can ever design a polytime algo
- It is "unlikely" . . . .

Given a graph $G=(V,E)$ is there a path/cycle that visits every vertex exactly once

Hamilton Path
Hamilton Cycle





No proof that no one can design efficient algorithm → lower bounds

> There is a class of very interesting natural problems for which polytime algorithms are not known

Further, there are other interesting properties

1 - If we can solve any of these problems efficiently $\Rightarrow$ efficient solns for the others also "reducibility"

(2) For these problems, the answer can be verified efficiently.

Prover	fast verification using "certificate"	Verifier

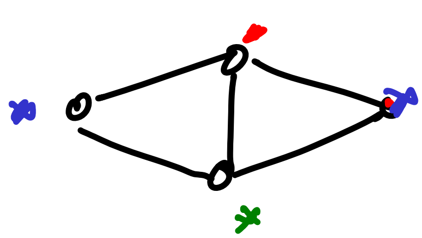
~~Non~~ deterministic Polynomial (NP)

Given a specific graph

This model is more suitable for decision problems (rather than optimization problems)

- Decision Knapsack : Is there a soln with profit $\geq P$?

- What is the min no. of colors required to "legally color a graph" (no two vertices with an edge can have the same color)



Can we color the graph using 4 colors?

Is the graph 100 colorable

Yes No

$\boxed{\text{Prover}}$ $\boxed{\text{Verifier}}$

produce a coloring using 100 colors

→ Is the graph not 100 colorable

The complement of the class NP is called co-NP.

and co-NP may not have an efficient verifier.