If "subtraction" is permitted then

\[[a, b] = [0, 5] - [0, a]\]

Various ways of partitioning \([0, b]\) as

union of the blue intervals

Goal: not use as few disjoint

intervals

\[[0, b'] = l_1' \cup l_5\]

Obs: We require at most one

interval from each level

(otherwise we use the larger interval for free.)
In general, for \( n \) points \((n = 2^k)\), we need at most \( k = O(\log n) \) "canonical" (blue) intervals suffice to express any arbitrary interval \([0, b]\).

How many blue intervals: \(2 + 2 + \cdots + n \leq 2n\).

For a general interval \([a, b]\), we can find the "split" point (pivot) that splits it into two "semintervals" and we can use the previous approach.

\( \Rightarrow \) \(2 \log n\) canonical intervals suffice for an arbitrary interval.
The binary tree structure storing the intervals is called a "range tree"