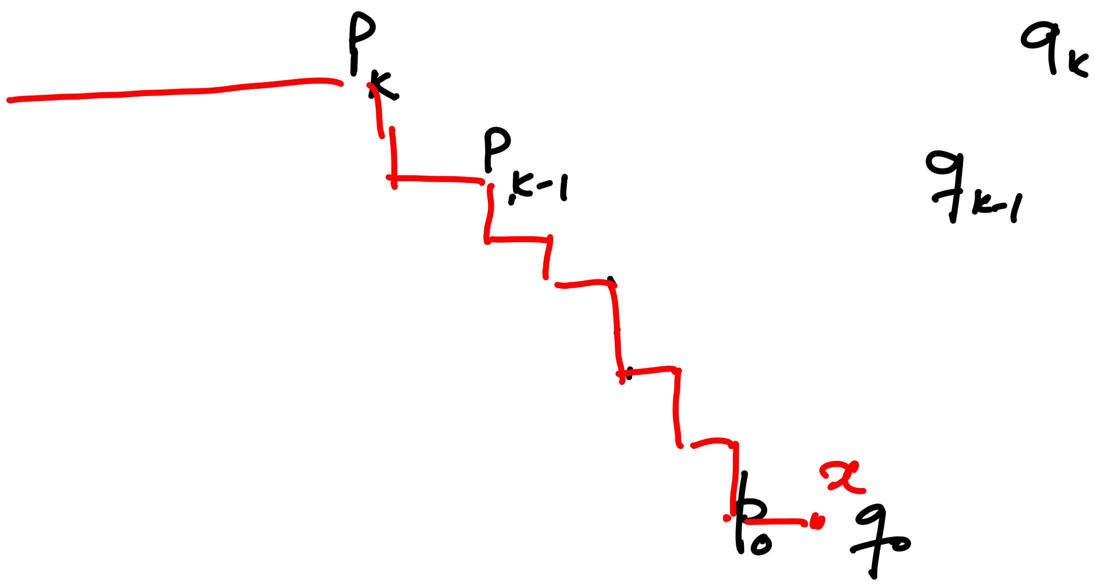
Lecture 21, CSL 630 Oct 16

Analysis of Skip Lists



$p_i \leq x \leq q_i$

The length of search path is the sum of the lengths of the search paths in each level

$$L = l_0 + l_1 + l_2 \dots l_k$$

$$E[L] = E[\sum l_i] = \sum_{i=1}^{k} E[l_i]$$

$E[l_i] = 2$ (# expected coin tosses to obtain the first head)

This calculation is not dependent on x, neither on the elements present in the Skip Lists

$E[L] = 2k$ (the # of horizontal steps)

vertical steps = k

What is k? (depends on how many nodes survive at each level)

2 ways

1

We try to bound the expected value of k (the top level has $\leq \alpha$ elements)

2

We choose k to be $c \log n$ and then try to bound the expected # elements in level k, say Y

Time for search $|L_k| + \dots$

1. What is the prob that an element is promoted to $c \log n$ levels, i.e. $k \geq c \log n$

$$\left(\frac{1}{2}\right)^{c \log n} = \frac{1}{n^c}$$

Suppose we choose $c = 2$

Then the prob that any of the n elements survive $c \log n$ levels $\leq \frac{1}{n^c} + \frac{1}{n^c} \dots n$ times (Union bound)

$\text{Prob}\left(\bigcup E_i\right) \leq \sum \text{Prob}(E_i)$

$$\leq \frac{1}{n}$$

If more than α elements survive $2\log n$ levels, then we rebuild the data structure (Approach 2)

$$E[X] = \sum_{i \geq 1} (\text{Prob } X \geq i)$$ for a discrete r.v. X

$$\text{So } E[K] = \sum_{i \geq 1} \sum_{1 \leq j \leq \log n - 1} \frac{1}{n^i} \cdot \frac{1}{2^j}$$

$$\leq O(\log n) + \log n \sum_{i \geq 2} \frac{1}{n^i} = O(\log n)$$

<u>Second approach</u>

$$Y_i = \begin{cases} 1 & \text{if } i^{th} \text{ element survives until } c\log n \text{ levels} \\ 0 & \text{otherwise} \end{cases}$$

$$Y = Y_1 + Y_2, \cdot Y_n$$

$$E[Y] = \sum E[Y_i] = \sum \text{prob that } i^{th} \text{ element survive}$$

$$= \sum_{i=1}^{n} \frac{1}{n^c} \ll 1$$

depending on c

Expected time for search is

expected $\#$ of elements in level $c\log n$

$+ 2 \cdot c\log n$

Concatenable queues (union and split)

can be implemented in

$O(\log n)$ time using a very simple

procedure