An alternate scheme for dynamic dictionary.

We have a sorted list

\[ L_0 \rightarrow 4 \rightarrow 9 \rightarrow 10 \rightarrow 15 \rightarrow 25 \rightarrow \ldots \rightarrow 100 \]

Search the sublist to find two elements, say \( p, q \) s.t. the element we are searching for, viz. \( x \)

\[ p \leq x \leq q \]

The time for refinement is \( \left| \left[ p, q \right] \cap L_0 \right| \)

\# elements \( \in L_0 \) in the interval \( \left[ p, q \right] \)

How do we search in \( L_1 \)? Recursive search in \( L \), using sublist \( L_2 \)
\[L_0 \leq \cdots \leq L_k = \{k\}\]

We maintain pointers between \(L_i\) and \(L_{i-1}\), so that we know which interval in \(L_i\) we must search given the position of \(x\) in \(L_i\). We obtain a sequence

\([p_k, q_k] \quad [p_{k-1}, q_{k-1}] \cdots [p_0, q_0]\)

\(p_i \leq x \leq q_i\)

Either \(x = p_i \Rightarrow x = q_i\) or there are the closest elements to \(x\) in \(L_i\)
The time to search is the cost of refinement in each level $i \left\lvert \left[ P_i, q_i \right] \cap L_{i-1} \right\rvert$. Then total time is $O(k)$.

Suppose this is constant.

We would like $k \to \log n$.

**Invariant**

If $L_i$ is chosen to be every alternate element of $L_{i-1}$ then cost of refinement is 2 and $k \to \log n$.

What happen when elements are added/added?

Picking every 2nd or 3rd element is too rigid and would be expensive to maintain dynamically.
Pick elements from \( L_{i-1} \) randomly (with prob \( \frac{1}{2} \)) to construct \( L_i \).

\[
\begin{array}{c}
L_i \\
\cdots \\
\hline
L_{i-1}
\end{array}
\]

How many elements in \( L_i \):

\[
E[|L_i|] = \frac{1}{2} |L_{i-1}|
\]

# copies of an element \( i \); the number (levels) of consecutive heads we obtain when we toss a fair coin

Say \( X_i \) is the # of copies of element \( i \);

\[
E[X_i] = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \cdots + i \cdot \left( \frac{1}{2} \right)^{i+1}
\]

\[
= 2
\]

Expected size of the data structure

\[
= 2 \times n
\]
What is the expected size of the "gap" $|[p_i, q_i] \cap L_{i-1}|$?