Function approximation

Given function in n steps

Minimize sum of squares: \( \sum_{i=1}^{n} (g(i) - f(i))^2 \)

Special case: \( g(i) \) is constant

Minimized when \( g = \frac{1}{n} \sum_{i=1}^{n} f(i) \)

Prove this.
Obs: For the optimal \( g^* \), the value at the \( k \)th step is average of the \( i_k \) values greater than \( i_k(?) \).

\( g \) is defined at \( i_1, i_2, \ldots, i_k \) where \( i_1, i_2, \ldots, i_k \in \{1 \ldots n\} \).

If we knew \( i_k \), we can define \( g^*(i_k) \).

Try all possibilities \( i_k \in \{1 \ldots n\} \), \( \{n-k \ldots n\} \) will suffice.

For each value of \( i_k \), we need an optimal \( g^* \) to approximate \( f \) between 1 .. \( i_k \).

\( g_{i,j} \) is the optimal \( i \) step function that approximates \( f(1) - f(2) \ldots f(j) \).

We are interested in \( g_{i,k,n} \).
\[ g^*_{i,j} = \frac{1}{d} \sum_{i=1}^{d} f(i) \] base case

Time : ?

\[ g^*_{i+1,j} : \text{Assume that we have computed } g^*_{i,j} \text{ for all } i \leq j \leq n \]

Find the optimal \((i+1)^{st}\) step for \(g^*_{i+1,j}\) using all possibilities between \(i\) and \(j\) and choose the best.

Let \(t(i,j)\) be the optimal \((i+1)^{st}\) step.

Then \(g^*_{i+1,j} = g^*_{i,t(i,j)} \frac{\text{average of values between } t(i,j) \text{ and } j}{j \leq n}\) we have from prior computation \(t(i,j)\) terms

In increasing \(i\) and for a fixed \(i \leq k \leq j \leq n\)

\(A(i,j)\) is the avg of value for \(i \leq j \leq i\)
If we compute $A(i,j)$ in advance
the running time is
\[ \sum_{i=1}^{k} \sum_{j=i}^{n} (j-i) \]
Total time.

\[ \sum_{i=1}^{k} \sum_{j=i}^{n} j = O(kn^2) \]
An edge labelled graph on some finite alphabet with \( |V| \) vertices and \( |E| \) edges. Each edge also has a \( \phi_\theta \) associated with it.

We want to find a path starting from \( v_0 \) with labels \( \sigma_0, \sigma_1, \ldots, \sigma_n \) \( \in \) alphabet such that \( \phi(v_0, v_1) \cdot \phi(v_1, v_2) \cdots \cdot \phi(v_n, v) \) is maximized.

If the string was length 1, we choose the edge with the correct label and mass prob
If we can solve the problem for lengths up to \( l \), then for length \( l+1 \) we can apply induction.