Scheduling problem

Given a set of n jobs J1, J2, ..., Jn with processing requirements 1 unit
and (integral) deadlines, we want to schedule them in a way so that
they get completed before deadlines.
If not, then job Ji incurs a penalty pi. Goal: Minimize penalty.

Eg. J1, J2, J3
    deadline 1 2 1
    penalty 5 2 8

    T | O | 1 | 2 | 3 | 4 |
    J3 | J2 | J1 |
    penalty: 5

Strategy: Earliest deadline first
break ties on the basis of penalty
Suppose we have a schedule $t_i \geq t_k \geq t_j \geq t_k \geq \ldots$

$d_i > d_k$ \[ \Rightarrow \]

If $d_i < t_k$ then we may incur penalty

Suppose the given schedule is "feasible" (no job incurs penalty) then

$d_k > t_k \quad d_i > d_k \Rightarrow d_i > t_k$

Objective: Minimize the penalty of the jobs that missed the deadline

\( \Rightarrow \) maximize the penalty of the "feasible" schedule

To apply "generic greedy" we must define "the subset system framework"
$S = \{ J_1, J_2, \ldots, J_n \}$

$J_i$: subset of $S$ that are "feasible" i.e. they can be scheduled without missing any deadline.

Moreover, any subset of a feasible set of Jobs is also feasible.

We would like to see if we can satisfy properties (2) or (3) of the mollard theorem.

**Exchange property.**

Given feasible sets $A$ and $B$ with $|B| > |A|$, can we add a job $j \in B - A$ to $A$ and still keep $A \cup \{j\}$ feasible?
Case 1: \( J_{k+1} \notin A \) and \( J_{k+1} \notin A \) 

Case 2: \( J_{k+1} \in A \Rightarrow J_{k+1} = J_i \)

Repeal the same argument with one job less in \( A \) and \( B \).

Either we terminate with case 1 or we are in a similar state where \( A = \emptyset \) and \( B \) has 1 job.

H.W.: Find a feasible schedule (the above argument gives a feasible set).

"Generic greedy works"
How about minimizing in the matroid framework?

Since $\emptyset$ is independent by defn $\emptyset$ is the min wt $= 0$

To deal with min. spanning trees, the underlying graph must be connected.

Then max. spanning forest is a max. spanning Tree.

Redefine the wt function as $W(e) = W_{\text{max}} - w(e)$

$W_{\text{max}}$ is the maximum wt of any edge.