CSL 630 Lecture 11 Sept 4

We have a ground set S of n elements. Let $\mathcal{M}$ be a family of subsets, $\mathcal{I} \subseteq 2^S$ (power set)

$\mathcal{I}$: are those subsets that are "feasible." We have a weight function $\omega : S \rightarrow R^+$

For a subset $P \subset S$

$$\omega(P) = \sum_{x \in P} \omega(x)$$

<u>Objective</u>: Find the largest weight subset from $\mathcal{I}$.

All (maximization) optimization problems can be formulated in this framework

<u>For example</u>: Knapsack problem

$S: \{x_1, x_2 \ldots x_n\}$

$\mathcal{I}$: all subsets of objects such that their weight is $\leq B$

$\underline{P \in \mathcal{I}}$ $\displaystyle\sum_{x_i \in P} v_i x_i \leq B$

<u>Maximize</u> : $\sum w(x)$

$w(x_i) = w_i$

Minimum Spanning Tree (Forest) $G = (V, E)$
(Maximum)

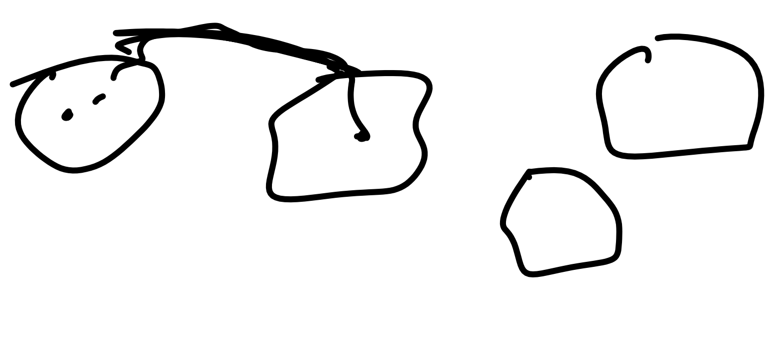
weight func w

S : set of edge E

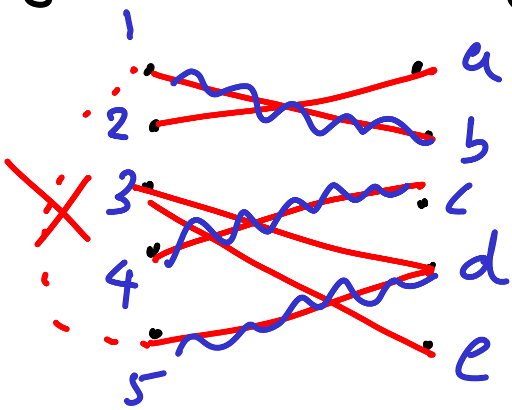
$\mathcal{I}$: all subsets of edges that do not have an induced cycle, i.e. all forests

<u>Objective fn</u> : Find the maxm weight forest.

<u>Obs</u> If the graph $G = (V, E)$ is connected then the soln is a tree (connected)



Eg. Matching in Graphs (bipartite)



Find a subset of edges such that no vertex has two edges incident

Maximum Cardinality matching (MCM)
Find a matching with maxm # edges.

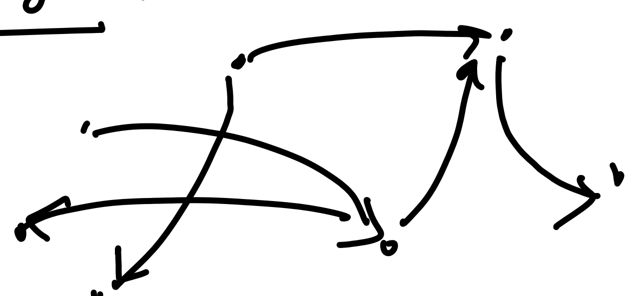
Maximum Weight Matching (MWM)
Each edge has a weight and we pick the matching with maxm weight

S : E $\quad$ $\mathcal{I}$: subsets of edges without common end-points

Obj function of MWM : pick the maxm wt subset from $\mathcal{I}$

Half matching :

Pick a subset of edges so that no vertex has more than one incoming edge



Generic greedy algorithm

S: ground set of elements $e_1, e_2, e_3 \ldots e_m$
$\mathcal{I}$: family of "independent" subsets
w: weight function $w: S \to R^+$
Objective. find a subset $P \subset S$, $P \in \mathcal{I}$
$w(P)$ is maxm.

Initialize $T = \phi$
Order the elements in decreasing sequence of weights (assume $w_1, w_2 \ldots$ is ordered) $O(m \log m)$

for $i = 1$ to m do
$\sum_i T(i)$
 if $T \cup \{e_i\} \in \mathcal{I}$ then
  $T \leftarrow T \cup \{e_i\}$

Output T

When is T the optimal soln?

Greedy succeeds for a problem if it solves all instances.

Subset system $M : (S, \mathcal{I})$

M is a subset system if

$\forall\ P \in \mathcal{I}, \quad Q \subset P \quad Q \in \mathcal{I}$

M is a matroid if

generic greedy solves the optimization problem for any weight function.

The following are equivalent

(1) $M = (S, \mathcal{I})$ is a matroid

(2) Exchange property: For all

$P, Q \in \mathcal{I}, \quad |P| > |Q|$

$\exists\ e \in P - Q$ s.t. $Q \cup \{e\} \in \mathcal{I}$

(3) Let $A \subseteq S$

Then all 'maximal' subsets of A have the same cardinality