CSL630 Lecture 10 Sept 1

0-1 Knapsack problem

Knapsack with some capacity, say B

Each of the n objects have a profit p_i and some weight w_i

Maximize $\sum p_i x_i$ $\quad x_i = \begin{cases} 1 \text{ if object } i \text{ is chosen} \\ 0 \text{ otherwise} \end{cases}$

s.t. $\sum x_i w_i \leq B$

$\boxed{w_i \leq B}$

$0 \leq x_i \leq 1$

Consider all possibilities i.e. all the 2^n possibilities of choosing the objects and choose the most profitable : Brute force / exhaustive

$$O(2^n \cdot n) \sim O(2^n)$$

There exists faster, polynomial time algorithms to solve linear prog (LP) (not simplex)

Choose the most profitable element among the remaining till we can't fit any more

$B = 15$

	x_1	x_2	x_3	x_4
profits	10	10	12	18
weights	2	4	6	9

$x_4 \ (18, 6)$ $\quad x_3 \ (18+12, 0)$

$\boxed{30}$

$\{x_1, x_2, x_4\}$

$\boxed{38}$,

No efficient polynomial time algorithm is known for 0-1 Knapsack.

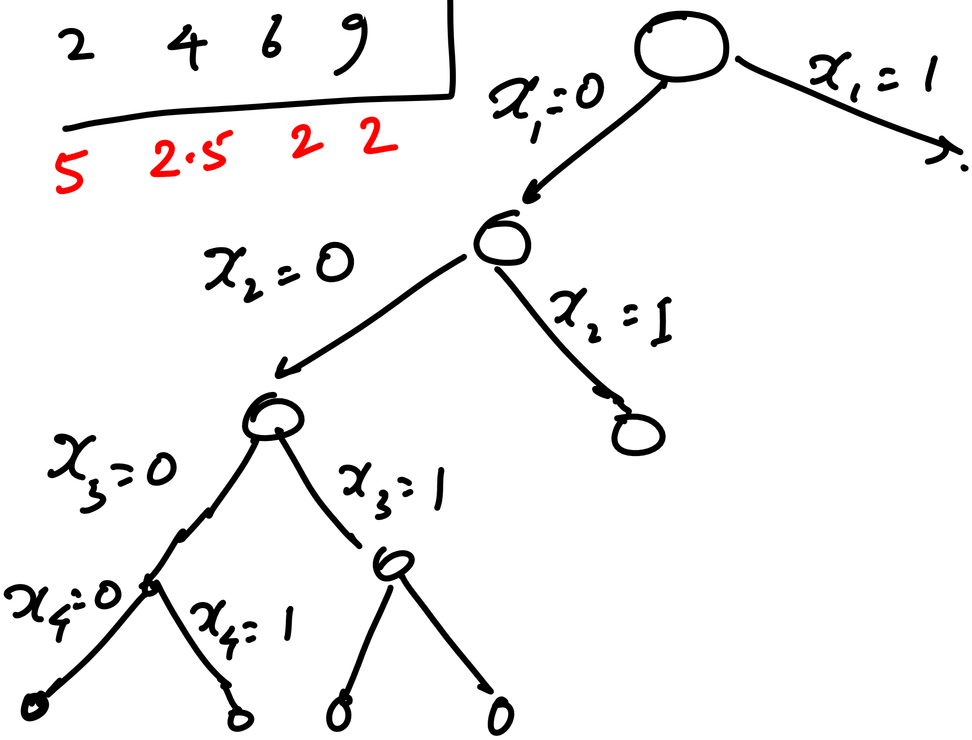
Use heuristics : and try to prove some properties about the heuristic even if it doesn't give the best soln

A very general heuristic

Branch and Bound

B = 15

10 ✓	10 ✓	12	18 ✓
2	4	6	9
5	2.5	2	2



If residual capacity = c and the max profit/wt = m
$\Rightarrow$ upperbound of $m \times c$
(better upper bounds welcome)

<u>Estimate function</u>: what is the best that we can get if we choose a subset of $\{x_{i_1}, x_{i_2} \ldots x_{i_k}\}$
(upper bound)

profit/weight ratio