1. Give an $O(n + m)$ time i.e. linear time algorithm to determine if a 2-SAT formula is satisfiable. The formula has $n$ variables and $m$ clauses. (Hint: $(x \lor y)$ is equivalent to $(\bar{x} \rightarrow y), (\bar{y} \rightarrow x)$.)

2. Prove that the following problems are NP-complete:

(a) **CLIQUE**

   *Instance:* Graphs $G=(V,E)$ and an integer $k \leq |V|$.
   *Question:* Is there a clique of size at least $k$ in $G$?

(b) **SUBGRAPH ISOMORPHISM**

   *Instance:* Graphs $G=(V,E)$ and $H=(V',E')$.
   *Question:* Is $H$ isomorphic to a subgraph of $G$?

   *Definition:* Graph $G_1 = (V_1, E_1)$ is **isomorphic** to $G_2 = (V_2, E_2)$ if there is a one-one onto function $f : V_1 \rightarrow V_2$ such that $(u, v) \in E_1$ iff $(f(u), f(v)) \in E_2$.

(b) **DOMINATING SET**

   *Instance:* Graph $G=(V,E)$ and positive integer $k$.
   *Question:* Is there $V' \subseteq V, |V'| = k$ such that each vertex $u$ in $(V-V')$ is adjacent to some vertex $v$ in $V'$.

3. (a) Show that **PARTITION** is self-reducible, i.e. give a polynomial time algorithm to solve the search problem, given a subroutine for the decision problem.

   (b) Show that **VERTEX COVER** and **CLIQUE** are self reducible.

4. Show the existence of a co-NP complete problem. Provide all formal definitions.