Reducing $3\text{SAT}$ to $V.C.$

$$(x_1 \lor x_3 \lor \overline{x_4}) \land (\overline{x_1} \lor x_2 \lor \overline{x_3}) \land \ldots$$

$m$ variables $x_1, x_2, \ldots, x_n$

$m$ clauses $C_1, C_2, \ldots, C_m$

$V = \{x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, \ldots, x_{n1}, x_{n2}, x_{n3}\}$

$E = \{(x_{11}, x_{12}), (x_{11}, x_{13}), (x_{12}, x_{13}), \ldots, (x_{n1}, x_{n2}), (x_{n2}, x_{n3}), (x_{n1}, x_{21}), (x_{12}, x_{23})\}$
Claim: The boolean formula $F$ is satisfiable if the graph $G = \phi(F)$ has a vertex cover of size $2m$ ($m$ is the number of clauses).

1st part: If $F$ is satisfiable, then $G$ has a VC of size $2m$.

To construct a cover of size $2m$, leave out one of the literals set to True and include the other two in the cover.

2nd part: If there is no VC of size $2m$, then $F$ is not satisfiable.

Truth assignment: There must be exactly 2 vertices in the cover for each clause $\Delta$. We assign the literal corresponding to the third vertex as True.
What is the smallest VC for a given graph?

**Observation**

If we could solve the decision problem, i.e., is there a VC of size $K$, then we can also solve the optimisation problem.

*Diagram:

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G
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**Maximal matching**: System has $K$ edges.

Then $V_i \cap V_j \geq K$

Choose both endpoints and call that subset $W$.

$W \subseteq V$
This is an approximation algorithm with approximation 2.

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\frac{\text{Size of our cover}}{\text{Size of optimal cover}} \leq 2
\]

→ Is there an approximation algorithm for VC with approx < 2?

→ It has been proved for many NP complete problems that approximation beyond a certain limit is not possible unless P = NP