Problem Given n jobs $J_1, J_2, \ldots, J_n$
and certain constraints $J_i < J_k$ to denote that $J_i$ must precede $J_k$, we want to find a feasible scheduling of n jobs or determine that it is not possible.

$J_A, J_B, J_C, J_D$

$J_A < J_B, J_A < J_D, J_D < J_C, J_C < J_B$

If the precedence graph contains a cycle $\Rightarrow$ not feasible.
We want to label the vertices of the precedence graph \( f: V \to \{1, 2, 3, \ldots, n\} \) so that
\[ \forall \bar{J}_i < J_k \quad f(i) < f(k) \]

If we do not have a cycle, is it always possible?

Is it always possible to number a DAG?

Using a simple induction on the number of vertices, and numbering a sink as \( n \), we can accomplish this.

A topological sort can be done if the directed graph has no cycles and can be done in \( O(m+n) \) steps.
\[ m = |E| \quad n = |V| \]
DFS numbering is a preorder numbering of the tree.

How do we use preorder/post order numbering of the DFS tree to accomplish topological sort?

**Claim**: If \( u \rightarrow v \), then the postorder \((v) > \text{postorder}(u)\) in a DAG.
A \not\rightarrow B \text{ does not imply } B \rightarrow A

Suppose \( A \rightarrow B \) and \( B \rightarrow A \)

If \( c \rightarrow A \) and \( A \rightarrow C \), is it true that \((C, B)\) is related?

Strongly connected component (SCC): A subset \( W \subseteq V \) s.t. \( x, y \in W, \ x \rightarrow y \) and \( y \rightarrow x \)
Component Graph is a DAG

Observation: The strongly connected components remain the same if we reverse the direction of every edge — call that graph $G^R$.

(The component graph also remains the same except the direction of edges.)
Claim: If we do a DFS on $G$, then one of the vertices in a "source" component will have the largest postorder number