Point location in planar partition

Point located involves 2 binary searches $O(\log n)$ time

Second binary search \( e_1 \) in based on above/below test in \( O(1) \) per test

\( S_i \), \( S_i', S_i'' \) - \( S_i' \)

\( e_1', e_2', e_1, e_2 \)

about below
Space is related to the search data structure in $2n+1$ vertical slabs.

Each such structure could involve $\Omega(n)$ segments $\Rightarrow O(n^2)$ space.

For e.g. in this kind of binary segments, each vertical slab can have about $\frac{n}{2}$ segments.
segment tree

each node represents a canonical cell of slab

elementary interval

segments completely spanning the slab
For point location, we need to do binary searches in vertical slabs that span the point exactly one per level.

Total time for log(n) binary searches (within the segment spanning the vertical slab).

Report the closest segment (distance to the segment) among the log(n) ray shooting queue.
The space corresponds to the total sizes of the segments stored in the vertical slabs.

Suppose $S(v)$ denotes the # of segments in vertical slab $v$.

$$\leq S(v) \leq \sum \text{N(s)}$$

For all nodes $v$, $s \in \text{Segment}$

$$\sum \# \text{ of Subsegments of } s$$

$$\leq 2n \log n$$

Ideally, we would like:

- $O(\log n)$ search time
- $O(n)$ space

Storing "similar lists" is done by "persistent data structures".
Constructing convex hulls

Convex hull of $P$, $CH(P)$, is the smallest convex set containing $P$.