Path found after

Maximum Weighted Matching (MWM)

Basic greedy fails since exchange property not satisfied

But the solution output by Greedy, say $G$, is at least $O/2$ where $O$ is the optimum solution

Let $O$ denote the set of edges that correspond to the optimal solution

$G \leq 0$

$(x, y) \in O$

$x \rightarrow w \rightarrow u \rightarrow v \rightarrow y$

$w_1 > w_2 > w$
\[(x, y) \cup y \in G \setminus O\]

We can do a counting argument between edges in \(G \setminus O\) and \(O \setminus G\).

In \(G \setminus O\):
- \(e_i \) "prevents" \(e'_i\).
- \(e_i \) is the first edge to stop \(e'_j\).

In \(O \setminus G\): \(w(e_i) > w(e'_i)\).

\[e_i \text{ can "prevent" at most two edges in } O \setminus G, \text{ say } e'_i, e''_i\]

\[\Rightarrow w(e_i) > \frac{w(e'_i) + w(e''_i)}{2}\]

So \(w(e_i) + w(e'_i) + \ldots + w(e_k) \geq \frac{w(e'_i) + w(e''_i) + w(e'_i) + w(e''_i)}{2} + \ldots + \frac{w(e'_i) + w(e''_i)}{2}\)

\[\Rightarrow w(\overline{O - G})\]

So \(w(G - O) + w(G \cap O) \geq w(\overline{O - G}) + w(\overline{G \cap O})\)

\[w(G) \geq \frac{w(\overline{O - G}) + w(\overline{G \cap O})}{2} + \frac{w(O)}{2}\]
Path Compression

Find(x)

All the nodes in the path $x \rightarrow \text{root}$ are made direct descendants of root. Overall effect is to bring a number of nodes closer to root within the same asymptotic complexity as $\text{Find}(x)$.

Note: Doesn't affect the properties on the rank.

The overall cost of $m$ finds is $O((m+n)\log^*n)$ using the path comp. & rank heuristic.
\[ \log^* (2) = 1 \]
\[ \log^* x = i \text{ if } \log(\log(\log x)) < 2 \]

\[ i \cdot \text{times} \]

E.g. \[ \log^* (2^2^1) = \log^* 2 + 1 = 2 \]
\[ \log^* (2^2^2) = \log^* 2 + 1 + 1 = 3 \.
\]
\[ \log^* (2^x) = \log^* x + 1 \]

A part of a family called inverse Ackerman function, the slowest growing function