Triangulation

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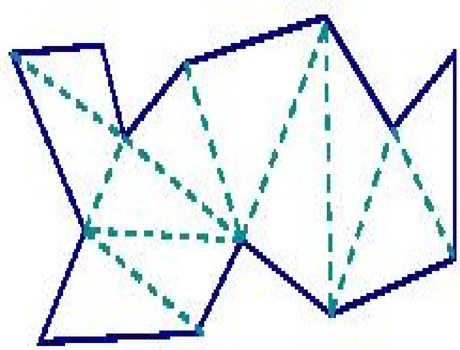
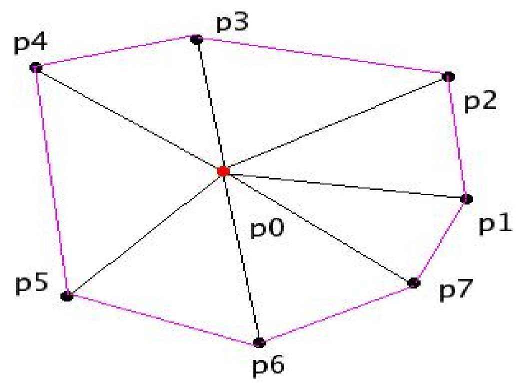
1 Definition



1.1 Polygon Triangulation

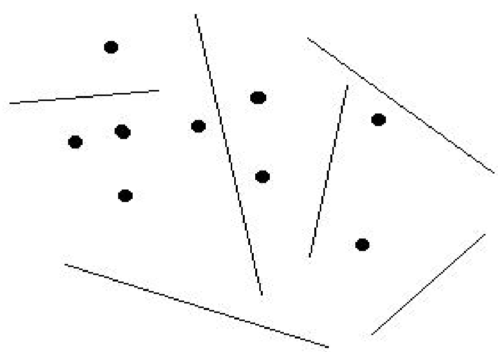
We have given a polygon without holes. We have to draw non-intersecting diagonals such that all the regions are triangles.

1.2 Points Triangulation



Given some points on the plane. To find non-intersectig lines joining these points such that all the regions are triangles and every point is a corner of atleast 1 triangle.

2 General Triangulation



Given some points and non-intersecting lines. Find new lines joining points and end points of lines such that :

- No two lines intersect.
- Every point is a corner of some triangle.
- Every line is an edge of some triangle.
- All regions are triangle except the exterior region.

3 Existence Proof

3.1 Polygon Triangulation

This claim has been considered under Ary Gallery Problem.

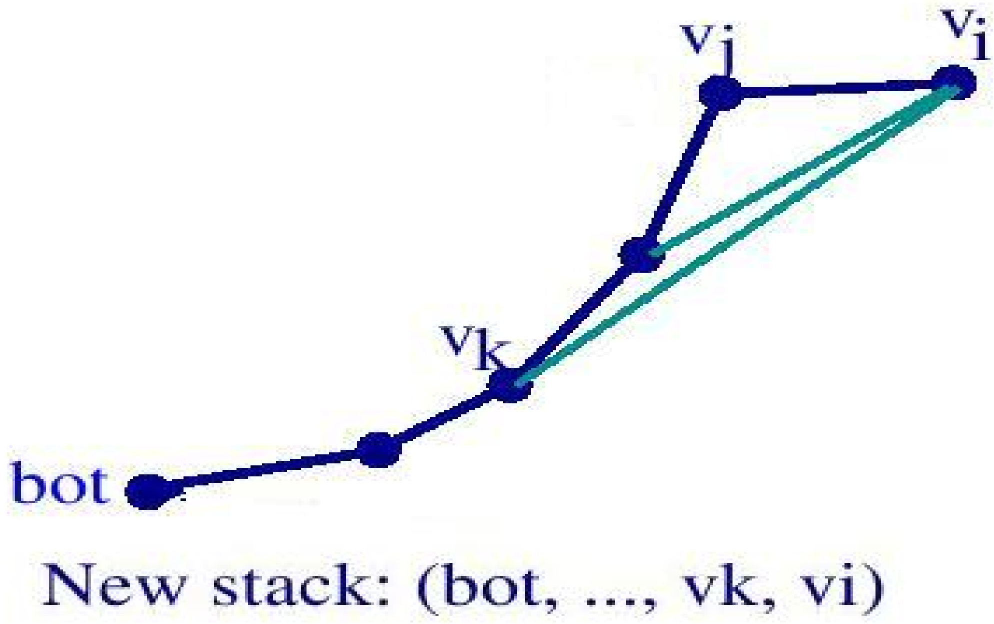
3.2 Point Triangulation

Pick a point from the given points (say p0). Join all points to p0. Sort the points radially. Join the consecutive points in sorted order. See the figure.

4 Monotone Polygon Triangulation

Monotone polygons are easy to triangualte as compared to general polygon. Following algorithm describes the monotone polygon triangulation.

1. Start from one end of the polygon.
2. Push next point(as ordered by the monotone chain) into the stack till the end of chain.
3. Check that the top 3 vertices can be triangulated(i.e. diagonal joining topmost and third topmost vertices lies inside the polygon) using right turn test. If yes, remove the second topmost vertex from the stack else go to step 2.
4. Repeat the above step.



Concave chain seem to cause problem but they will reduce to convex chain.
Running Time of this algorithm is O(n).

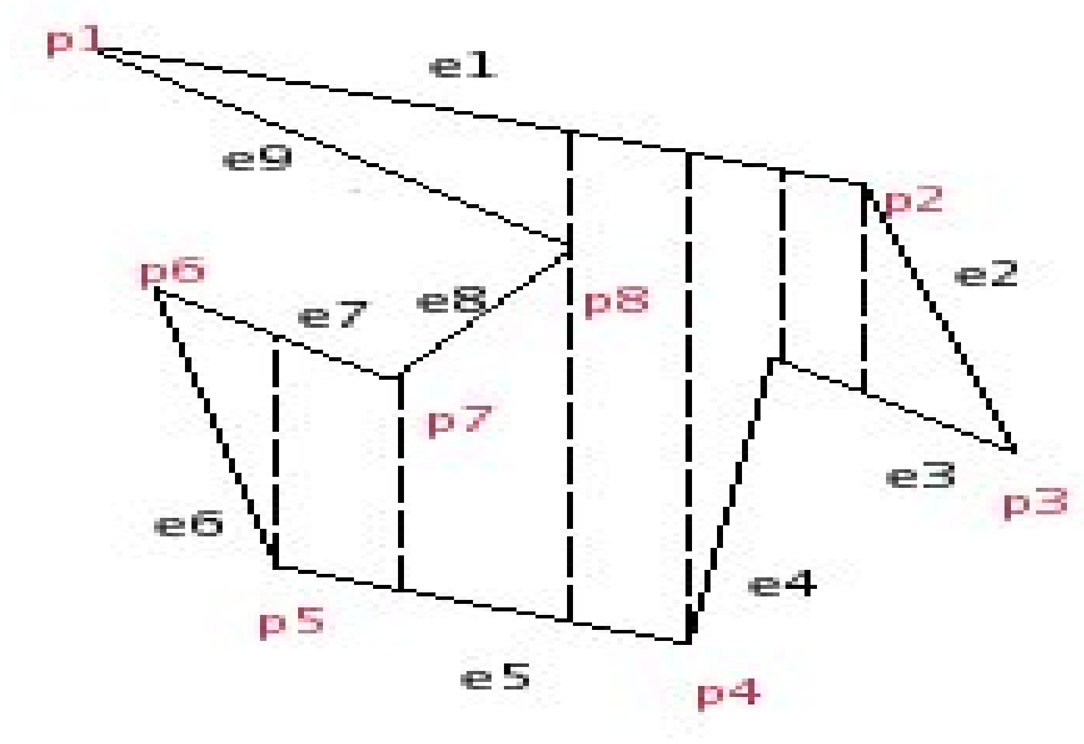
5 General Polygon Triangulation

General polygon triangulation proceeds in three steps:

- Trapezoidal decomposition using plain sweep algorithm. This will take O(nlogn) time as described later on.
- Construction of monotone polygons from trapezoidal decomposition. This step will take O(n) time as described in the later section.
- Apply monotone polygon triangulation method to the sub-problems. Monotone triangulation for all the sub-problems will take O(n) time as described in previous section.

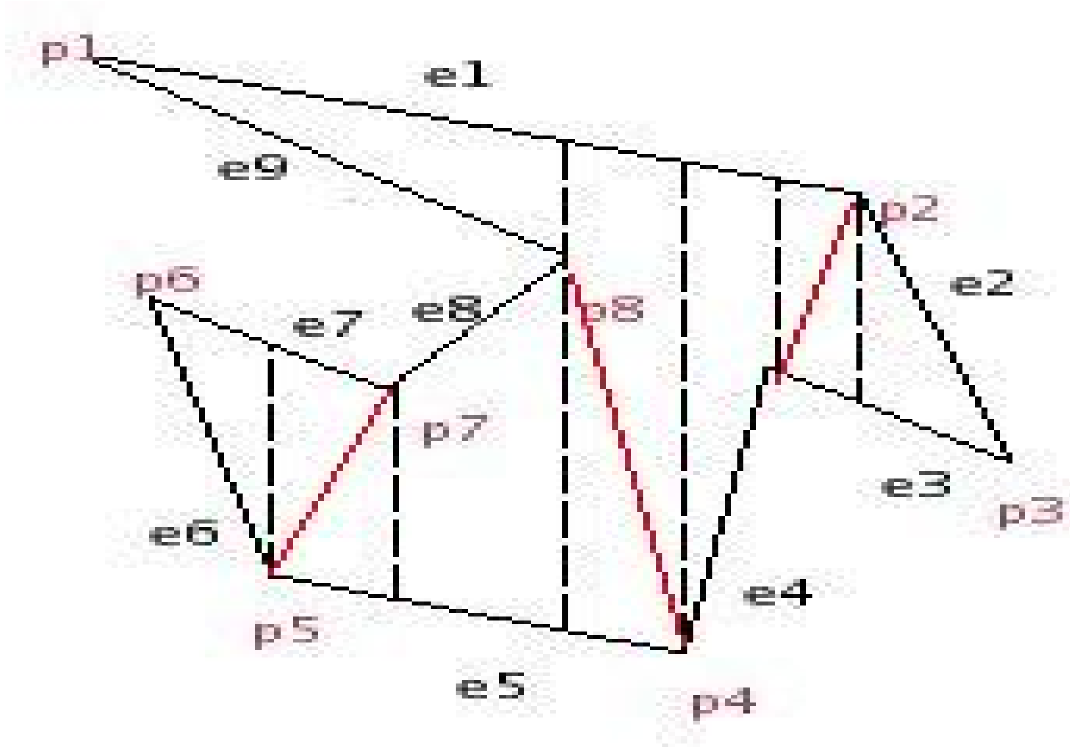
So Overall running time for the above procedure is O(nlogn). In the following section we will discuss first two steps of the procedure.

5.1 Trapezoidal Decomposition



Trapezoidal Decomposition can be done using line sweep algorithm as described in the class. Running time for this method is O(nlogn).

5.2 Monotone Polygon Construction



With given trapezoidal decomposition we can decompose the given polygon into the monotone polygons in O(n) time as described below.
For every edge : Find all the corner points that are projecting on this edge using trapezoidal decomposition. All these corner points and edge form x-monotone polygons. There are some edges that don't have any point projecting on them. These edges will not construct any subproblem. Projection also includes projection of points on above edges.
Claim : These polygons are monotone and non-intersecting.
Running Time for this method is O(n).