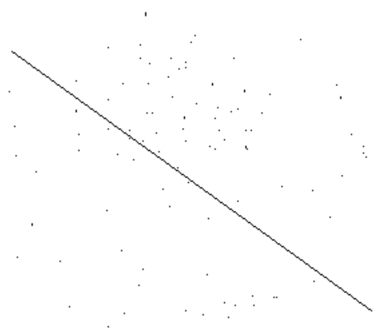
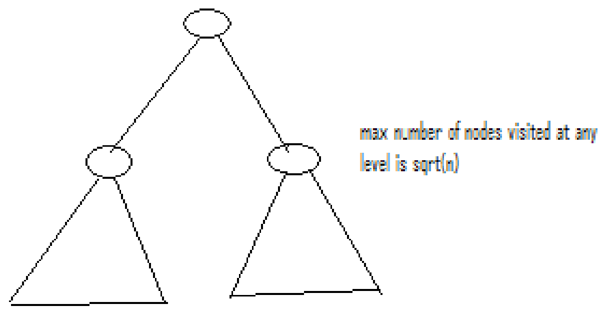
Partition Problem



Problem:Given n points in space and given a query line l we have to report the points which lie below the query line l.

Make a convex hull of the given set of points.Find the intersection of line l with the convex hull.If the line doesn't intersects the hull then take any point on the hull and find if it lies above or below the line l and thus conclude that none or all points lie below l respectively.

If the line intersects the hull then divide the set into parts and make convex hull of both the parts and again check the above condition for line and hull.Thus build a binary tree for this recursive algorithm.It has been poproposed that atmost O($n^{\frac{1}{2}}$) nodes are there at any level.Thus the number of nodes to be visited is O($n^{\frac{1}{2}}$

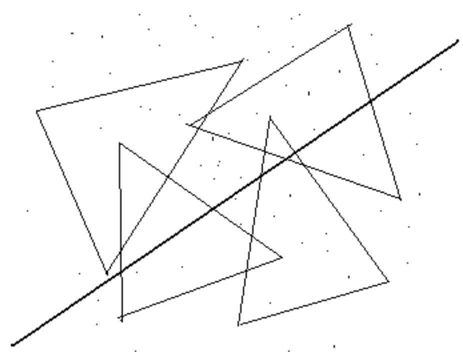


$\log n$)

Query Time: The total query time is O($\log n$) time the number of nodes visited which is O($n^{\frac{1}{2}} \log n^2$)

Space: Solving the recurrence T(n)=n + 2T($\frac{n}{2}$) the space requirement comes out to be O(n $\log n$).

Open problem:Show that for any point set P in the plane $\ni$ a tree T such that a line l crosses at most $w_l^{\frac{1}{2}}$ $\log \frac{n}{w_l}$ edges of T , where w_l is weight of l.



Simplicial partition: A set of triangles in the given space of points such that each point is in atleast one triangle.

Crossing number: Maximum number of triangles intersected by a line.

Then $\ni$ a fine simplicial partition of size r whose crossing number is O($r^{\frac{1}{2}}$). This is another solution of the above problem where the the space containing n points is divided into r triangles such that they form a simplicial partition.Now to report the number of points below a given query line we check the intersection of query line with each triangle.

Now it is proposed that the line can intersect only $r^{\frac{1}{2}}$ trinagles and each triangle can have atmost $\frac{2n}{r}$ points.Now with each trinagle it intersect we again apply the same rule treating this triangle as our new space.Thus we have:

Query time: Q(n)=r + $r^{\frac{1}{2}}$ Q($\frac{2n}{r}$) which solves to O($n^{\frac{1}{2}}$ $(\log n)^2$)

Space: M(n)= n + $r^{\frac{1}{2}}$ M($\frac{2n}{r}$) which solves to O(nlog n)