Computational Geometry Lecture 7

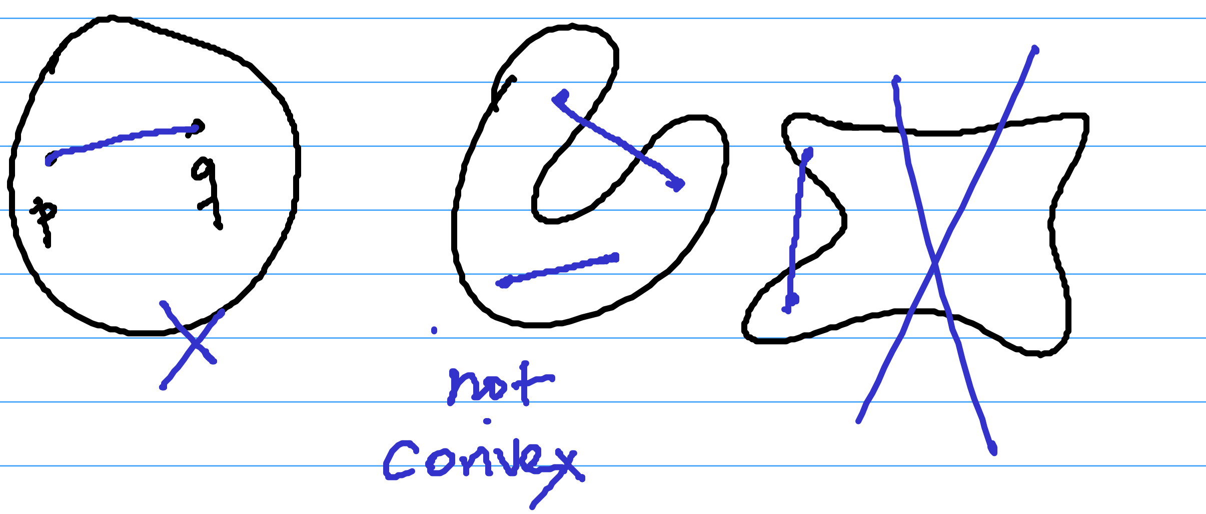
Topic Convex hull

Convex set : $X \subseteq \mathbb{R}^2$ satisfies the following property

for any two points $p, q \in X$ the convex linear combination, i.e.

$$\lambda \cdot p + (1-\lambda) q \in X \quad 0 < \lambda < 1$$

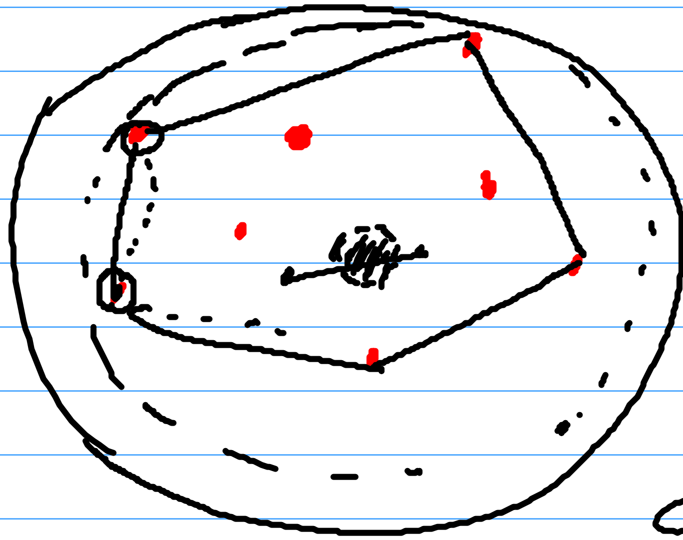
(the entire segment $\overline{pq} \subset X$



Convex hull (S) S is a given set of objects

Suppose S is a finite set of points

The convex hull of S is the smallest convex set that contains S.



It is a convex polygon

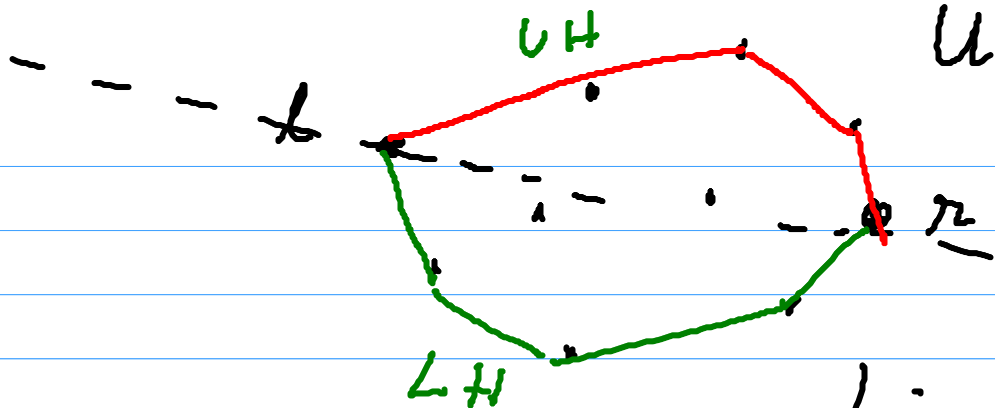
- It can be proved that the CH(S) is the smallest convex polygon containing S in terms of (i) area (ii) perimeter

Some points are on the boundary of the convex hull : corner points / boundary points

The boundary of CH(S) is an ordered chain (of vertices or edges)

Given a set S of n points, how do construct CH(S) ?

A boundary point supports a tangent i.e., a line that passes thru the point keeping the remaining points on one side.



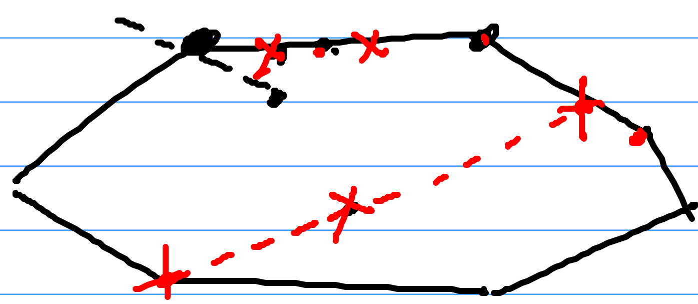
U: all points above the line

L: all points below the line

How do we construct the upperhull?

Boundary point: (alternate defn)

A point $p \in S$ that cannot be expressed as a clc of two points in the convex hull of S.



No point that is a clc of two other points in CH(S) is a corner point.

Computational Geometry Lecture 8

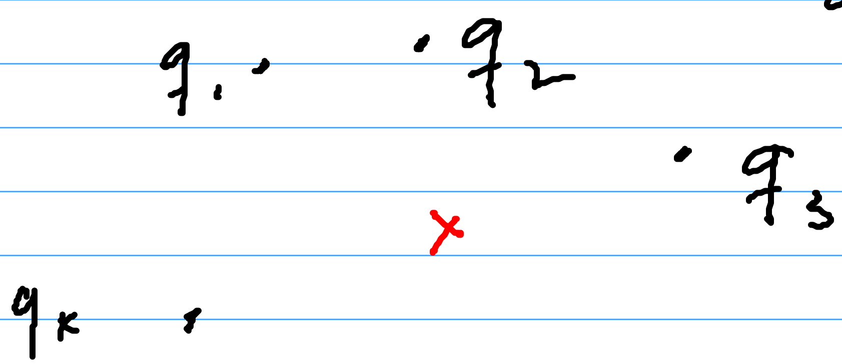
Topic : Convex hulls – continued

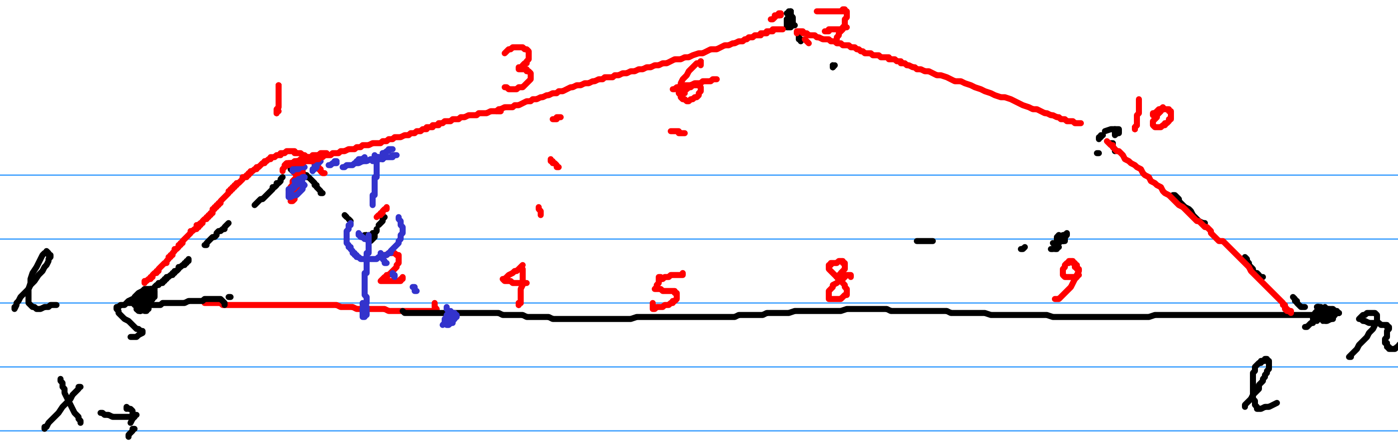
A point p in a clc of points

$$q_1, q_2, \ldots q_k$$

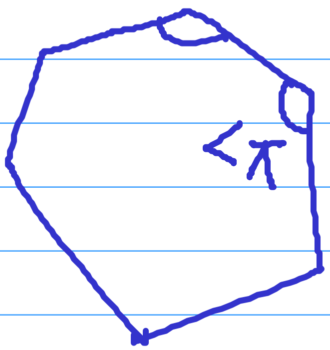
if $$p = \lambda_1 q_1 + \lambda_2 q_2 + \cdot\cdot \lambda_k q_k$$

$$0 < \lambda_i < 1 \quad \sum_i \lambda_i = 1$$





Monotone chain



$<\pi$: convex angle

$>\pi$ reflex angle

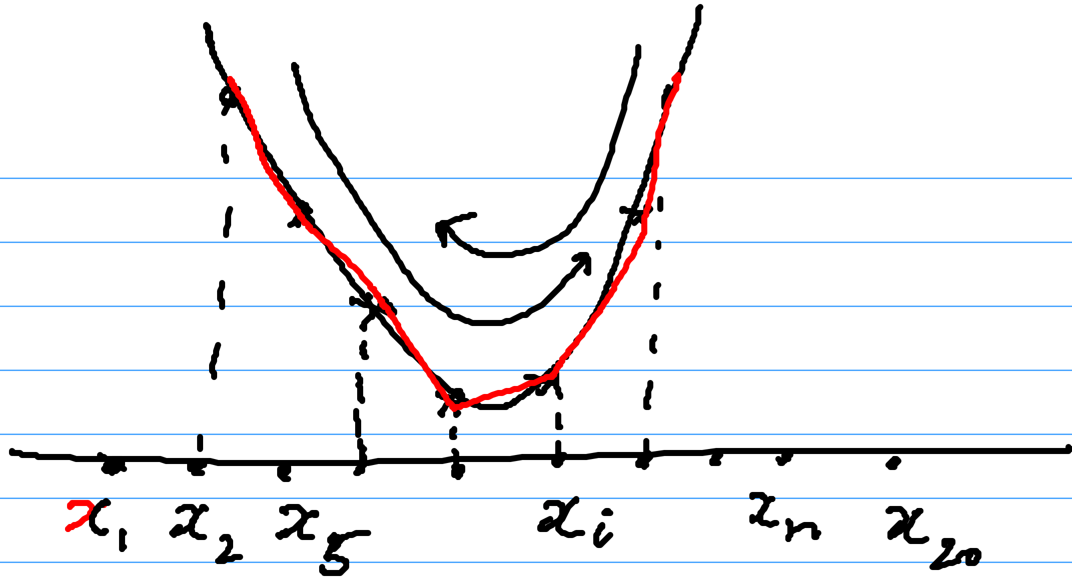
Running Time

- Sorting : $O(n \log n)$
- Constructing a convex chain by adding the next point or deleting ..? points

Since a point can be added or deleted once, using a stack, running time (# left-turn / right turns) is $O(n)$.

Graham scan

Can we do better?



$x_i \rightarrow (x_i, x_i^2)$ Given $S = \{x_1, x_2 .. x_n\}$

$S' = \{(x_1, x_1^2)(x_2, x_2^2). \}$

CH(S') gives us an ordered set of boundary points so that we can sort S using this ordering

$$\text{Sorting}(S) \propto_{O(n)} CH(S')$$

$\therefore$ CH must have a lower bound of Sorting

$$P_1 \propto_{f(n)} P_2$$

lbnd of P_1 is applicable P_2

and the ubnd of P_2 is " to P_1

Observations on the lowerbound proof

1. We are assuming that CH algo must output ordered set of corner vertices.

(instead of outputting the corner points in any order).

2. If all points are not on the boundary, i.e. only $h << n$

??

How about $\underline{O(n \log h)}$?

nh

$n \log n$

Quickhull

Running Time