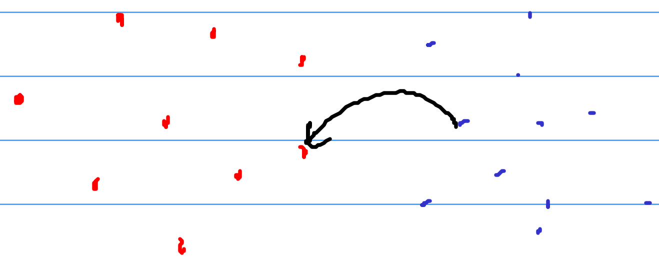
Lecture 40: Shape Comparison



R B $|R| = |B| = n$

$\longrightarrow Q : B \longrightarrow R$

Directed Hausdorff distance

$H(B,R)$ $\max_{b \in B} \|b - Q(b)\|$: L_∞-norm

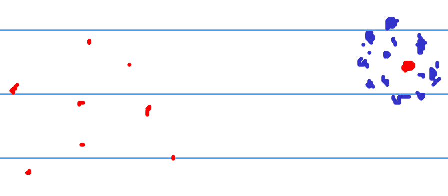
$\mu(R,B)$

$\sum_{b \in B} \|b - Q(b)\|$: L_1-norm

$\sum_{b \in B} \|b - Q(b)\|^2$ L_2-norm

$Q(b)$: Nearest neighbor of b in R

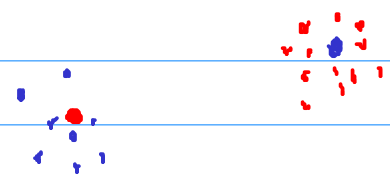
$O(n \log n)$ time in $\mathbb{R}^2$



$$H(B,R) = \max_{b\in B} \underbrace{\min_{r\in R} \|b-r\|}_{\|b-Q(b)\|}$$

Hausdorff distance

$$h(R,B) = \max\{H(B,R), H(R,B)\}$$



Q ! bijective

1-1 mapping between R & B

Minimum weight matching between R & B.

$G = (V,E)$: $w: E \to \mathbb{R}^+$

Matching: $M \subseteq E$: vertex disjoint edges

$G = (R \cup B, R \times B)$ bipartite graph

$w(r,b) = \|r-b\|$

$O(mn)$

$O(n^3)$ algorithm $m \geq n^2$

For $(R \cup B, R \times B)$:

best-known algorithm takes $\tilde{O}(n^2)$ time

Is there a subquadratic algorithm?

$\sim n^{1+1/c}$ time

$\log c$ approximation !

$O(1)$ factor - estimate the cost in $O(n \log^2 n)$

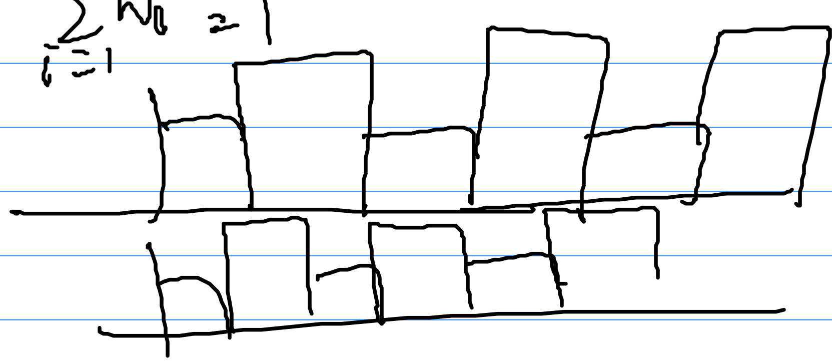
Earth Mover's distance (transportation distance)

$R, \quad B = \{b_1, \dots b_n\}$

$\{r_1, \dots, r_n\} \quad w_1' \dots w_n'$

$w_1 \dots w_n \quad \sum_{i=1}^{n} w_i' = 1$

$\sum_{i=1}^{n} w_i = 1$



R, B and given μ

Find Π: set of all rigid motions

Compute

$$\min_{\pi \in \Pi} \mu(R, \pi(B))$$

ICP: Iterative Closest Pair Algorithm

B_i: current position of B

R

compute $Q(b)$ for all $b \in B_i$

is fixed

Translate B_i s.t.

$$\pi^* = \overset{B_i}{\arg} \min_{\pi \in \Pi} \sum_{b \in B} \| b - Q(b) + t \|^2$$

$$B_{i+1} = \pi^*(B_i)$$