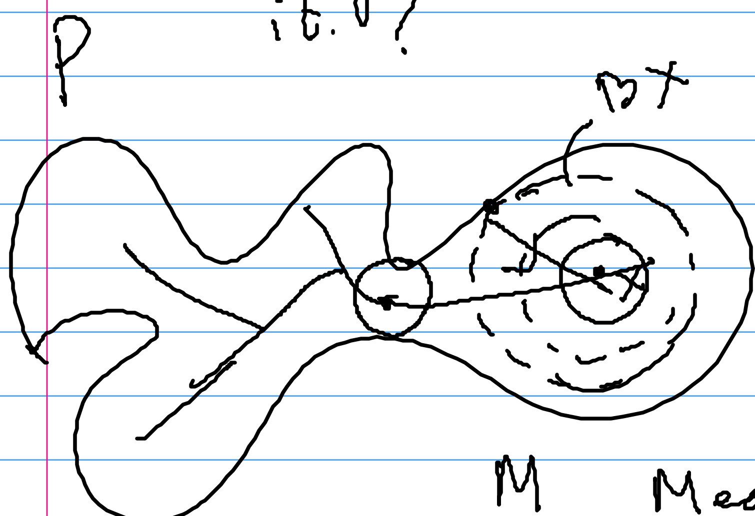
Lecture 39: Shape Representation

Sampling

How densely one should sample a geometric shape to reconstruct it?



- Medial axis
- Local feature size.

B_x: smallest ball at x that touches ∂P.

M Medial axis of P

$$M = \{x \mid |B_x \cap \partial P| > 1\}$$

If ∂P is connected, Medial axis of P is a tree.

Retraction $Q: \partial P \longrightarrow M$

$$Q(p) = x \text{ if } p \in B_x \cap \partial P$$

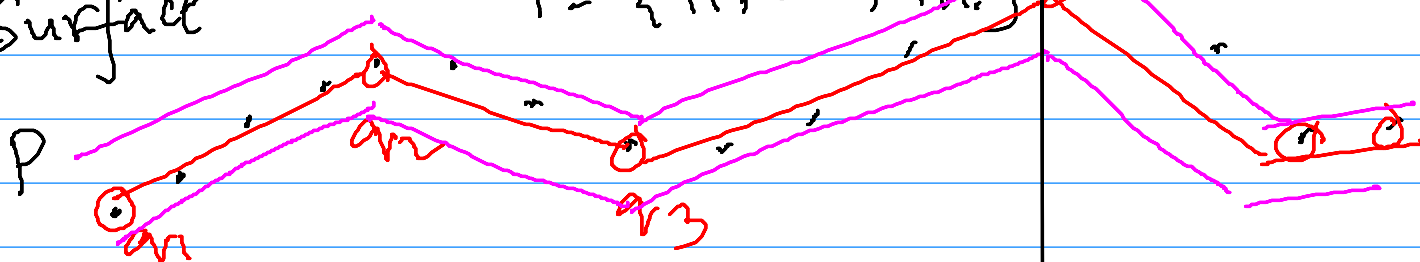
$$LFS(p) = \| p\, Q(p) \|$$

Sampling density around p $\propto \dfrac{1}{LFS(p)}$

h

Curve / Surface Simplification

$P = \{p_1, \dots, p_n\}$



ε: tolerance error

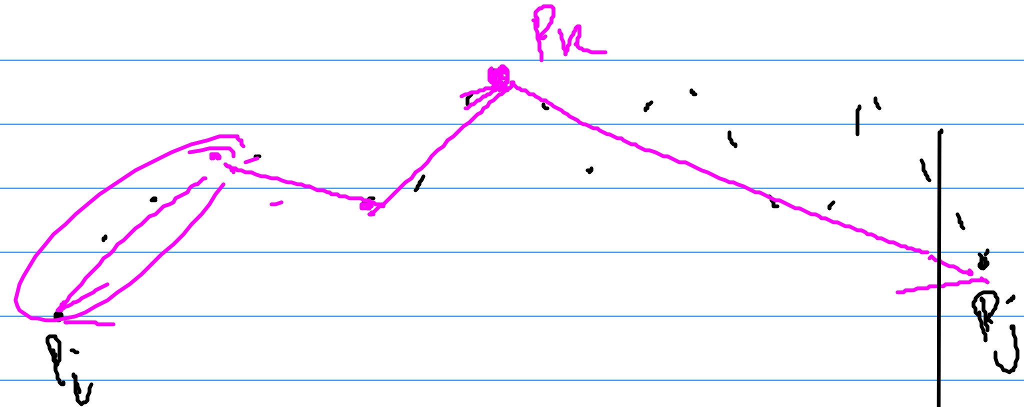
$Q \subseteq P$ $\quad Q = \{q_1, \dots, q_k\}$

each p_i lies within distance ε from the chain $q_1 q_2 \cdots q_k$

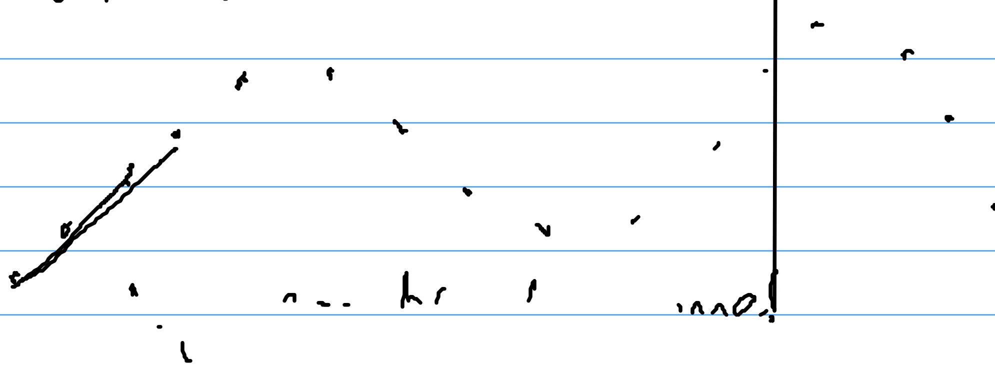
k should be small.

- Refinement
- Decimation

Douglas-Peucker algorithm

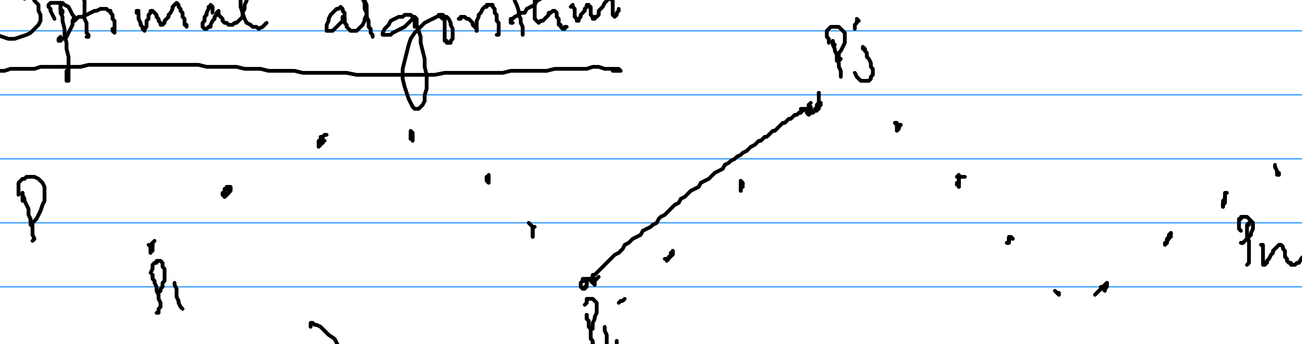


Decimation Method



Edge contraction method.

Optimal algorithm



$G = (P, E)$

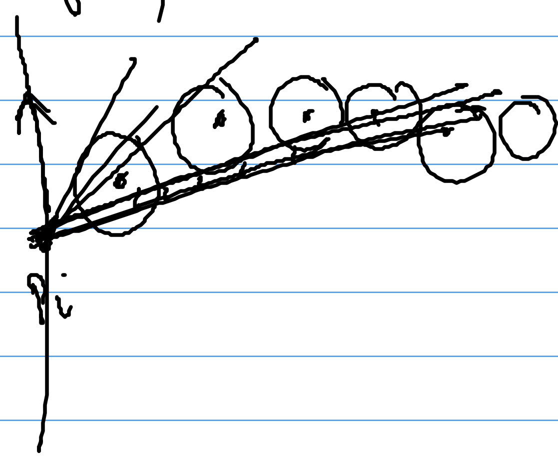
$(p_i, p_j) \in E$ if

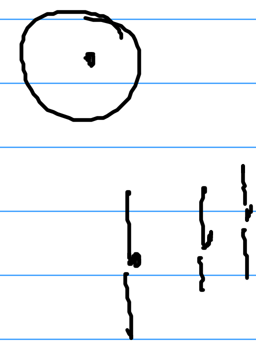
$d(p_k, \overline{p_i p_j}) \leq \varepsilon \quad \forall\, i \leq k \leq j$

Find the shortest path from p_1 to p_n in G.

$O(n^3)$: straightforward.

$O(n^2 \log n)$

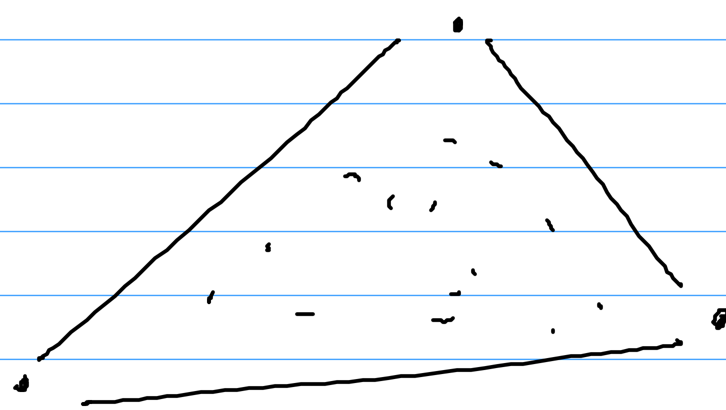




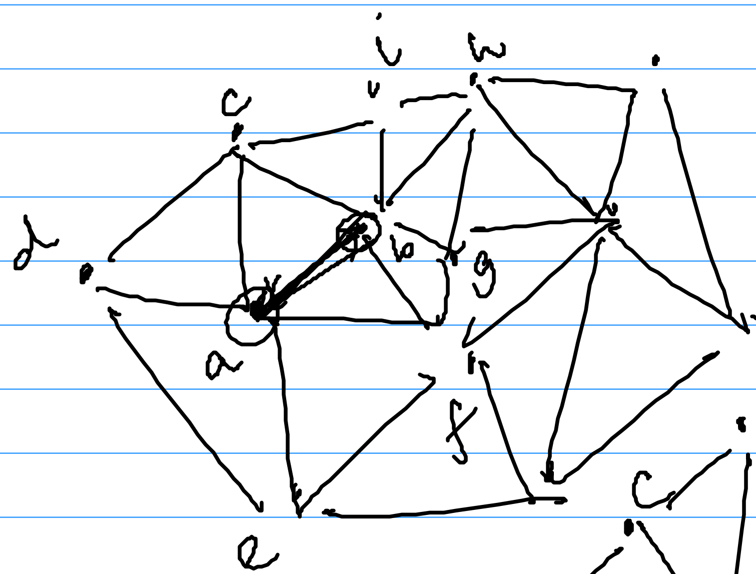
$n^{4/3}$

Chebyshev metric

Surface Simplification



Decimation method:



Garland-
Heckbert
algorithm