Computational Geometry Lecture 34

Topic : ε-WSPD construction contd.

By the charging argument, where an output pair (u_s, v_s) is charged to $v' =$ parent of v_s

where $(u_{s-1}, v_{s-1}) \longrightarrow (u_s, v_s)$

and $u_s = u_{s-1}$

v_{s-1} is the parent of v_s in T

a maximum of $O\left(\frac{2^{2d}}{\varepsilon^d}\right)$ changes can accumulate in any node of T

$\Rightarrow$ Total number of output pairs is

$$O\left(n \cdot \frac{1}{(\varepsilon)^d}\right) \text{ for } \varepsilon > 0$$

and fixed d.

Total construction time is $O\left(n \log n + n \cdot \frac{1}{\varepsilon^d}\right)$

Given an ϵ - WSPD of point set S, construct a $(1+\delta)$ spanner of S.

A spanner G_S of point set S is a subgraph of the complete graph on S ($\binom{n}{2}$ edges) where each edge (p_1, p_2) has wt $= \|p_1 - p_2\|$

$p_1, p_2 \in S$.

$(1+\delta)$ spanner guarantees that

$$d_G(p_1, p_2) \leq (1+\delta) \cdot \|p_1 - p_2\|$$

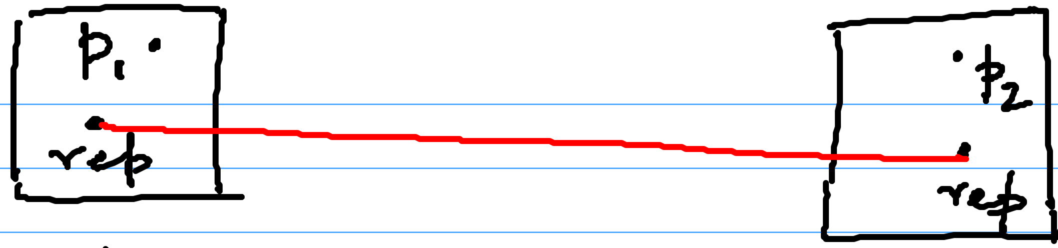
for any specified $\delta > 0$

Given a ϵ- WSPD of S
(for a suitably chosen ϵ, depending on δ), we can construct
a $1+\delta$ Spanner using the following edges.

$$(A_1, B_1), (A_2, B_2) \cdots (A_t, B_t)$$

Choose a represtative $rep(A_i)$
and " " $rep(B_i)$
and add the edge $(rep(A_i), rep(B_i))$

edges in the $1+\delta$ spanner
= # edges of ϵ- WSPD
$O\left(\frac{n}{\epsilon^d}\right)$ edges



$A_i = A$ $\qquad\qquad$ $B_i = B$

Suppose (by induction) that all pairs (x, y) such that $\|x - y\| < \|p_1 - p_2\|$ the

$$d_G(x, y) \leq (1 + \delta)\|x - y\|$$

$$\|rep(A) - rep(B)\| \leq \|p_1 - rep(A)\| + \|p_2 - rep(B)\| + \|p_1 - p_2\|$$

by Δ inequality

$$\|p_1 - rep(A)\| \leq \square_A \leq \epsilon \cdot d(\square_A, \square_B) \leq \epsilon \cdot \|p_1 - p_2\|$$

Similarly $\|p_2 - rep(B)\| \leq \epsilon \cdot \|p_1 - p_2\|$

So $\|rep(A) - rep(B)\| \leq (1 + 2\epsilon)(\|p_1 - p_2\|)$

From I.H.,

$$d_G(p_1, rep(A)) \leqslant (1+\delta)\|p_1 - rep(A)\|$$

$$\leqslant 1+\delta \cdot \square_A \leqslant (1+\delta)\varepsilon \cdot \|p_1 - p_2\|$$

since $\|p_1 - p_2\| \geqslant d(\square_A, \square_B)$

Similarly

$$d_G(p_2, rep(B)) \leqslant (1+\delta)\cdot\varepsilon\cdot\|p_1 - p_2\|.$$

$$\text{So } d_G(p_1, p_2) \leqslant d_G(p_1, rep(A)) + d_G(p_2, rep(B)) + \underbrace{d_G(rep(A), rep(B))}_{= \|rep(A) - rep(B)\|}$$

$$\leqslant (1+\delta)\cdot\varepsilon\cdot\|p_1 - p_2\| + (1+\delta)\cdot\varepsilon\cdot\|p_1 - p_2\| + \underbrace{rep(A) - rep(B)}_{\leqslant (1+2\varepsilon)(\|p_1 - p_2\|}$$

So R.H.S.

$$\leqslant \Big(\underbrace{2(1+\delta)\cdot\varepsilon + 1+2\varepsilon}_{1+4\varepsilon+2\varepsilon\delta}\Big)\|p_1 - p_2\|$$

Choose ε, s.t. $4\varepsilon + 2\varepsilon\delta \leqslant \delta$