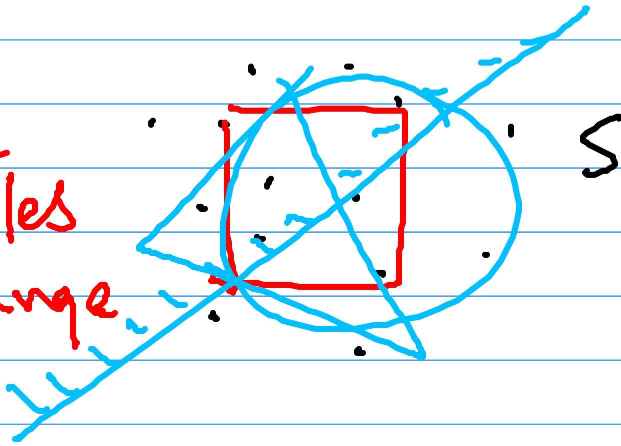
Lect. #31 : Range Searching

$(S, \mathcal{R})$



$\mathcal{R}$: set of orthogonal rectangles

Orthogonal range searching

set of triangles, disks, halfplanes, ...

$(X, \oplus)$: semigroup

$w : S \longrightarrow X$

Preprocess S into a data structure s.t. for a query region $r \in \mathcal{R}$, compute

$$\bigoplus_{p \in S \cap r} \qquad \sum_{p \in S \cap r} w(p)$$

Range counting $\quad X = \mathbb{N} \quad \oplus = +$

$w(p) = 1 \quad \forall p \in S$

Emptiness query $\quad$ yes if $r \cap S \neq \emptyset$

$X = \{0, 1\} \quad \oplus = \vee$

Range Reporting

$$X = 2^S \quad + = \cup$$

$$w(p) = \{p\}$$

Orthogonal

$Q(n)$: Query time $\quad O(\sqrt{n}) \quad O(\log n)$ RS

$S(n)$: Size $\quad O(n) \quad O(n \log n)$

$P(n)$: Preprocessing time

- Semigroup model

$\mathcal{F} = \{C_1, \ldots, C_m\}$: canonical subsets

- $C_i \subseteq S$
- $\forall r \in \mathcal{R}$

$$\mathcal{F} \supseteq \mathcal{F}(r) = \{C_1, \ldots, C_u\}$$

$$* \; C_i \cap C_j = \phi \quad \forall C_i, C_j \in \mathcal{F}(r)$$

$$* \; \bigcup_{C_i \in \mathcal{F}(r)} C_i = S \cap r$$

Minimize $\quad$ Size $\geq m \quad Q(n) \geq \max_{r \in \mathcal{R}} |\mathcal{F}(r)|$

Filtering search

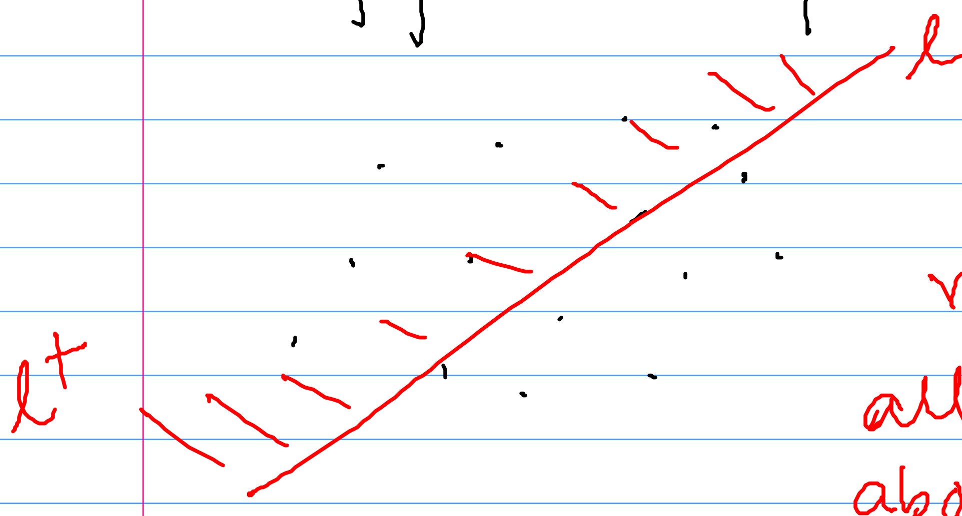
Range reporting

k: O/P size

$$Q(n) = \Omega(k)$$

Query procedure can use $O(k)$ time to guide the search.

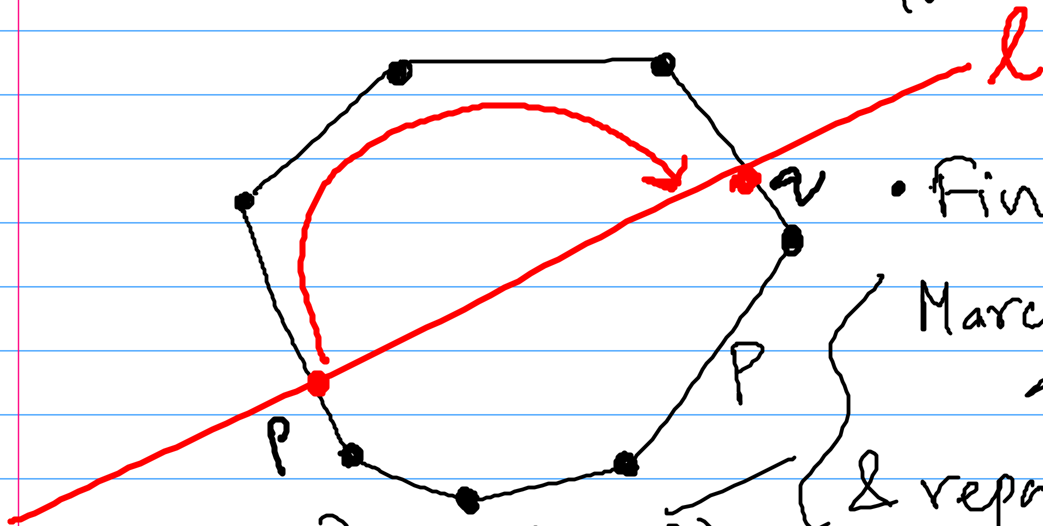
Halfplane Range Reporting



Given a query line ℓ report $S \cap \ell^+$, all points that lie above ℓ.

Suppose points in S are in convex position

Intersect(P, ℓ)



- Find $p, q = \ell \cap P$

  March from p to q along P & report all points in $\ell^+ \cap S$.

Report(p, q, P)

Time $= O(\log n + k)$

- Compute convex layers $P_0, P_1, \ldots, P_m$ of S

  $S \supseteq S_i$: set of points on P_i

  Size $= O(n)$ $\quad P(n) = O(n \log n)$

- Query procedure

```
i = 0
while p,q = Intersect(ℓ, P_i)
    Report(p, q, P_i)
    i = i+1
end while
```

Query time:

Suppose ℓ intersects $P_0 \ldots P_r$ & doesn't intersect P_{r+1}

$$\underbrace{(r+1) \cdot \log n}_{=} + k$$

$$r \le k$$

$$\boxed{Q(n) = O((k+1) \cdot \log n)}$$

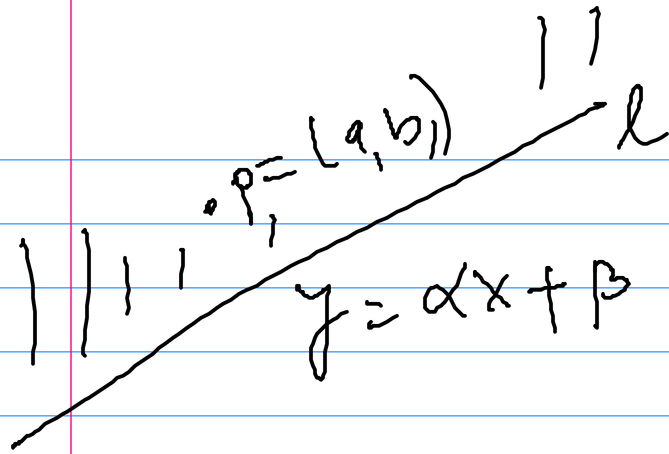
Fractional cascading

$$O(\log n + k)$$

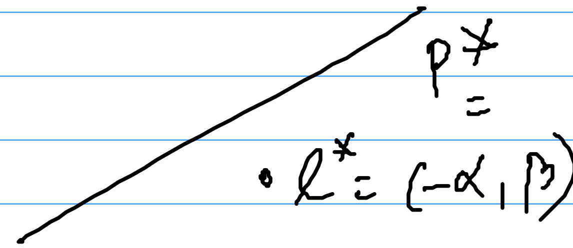
Halfplane Range Counting

$Q(n) = O(\log n)$ $S(n) = O(n^2)$

$Q(n) = O(\sqrt{n} \log n)$ $S(n) = O(n)$



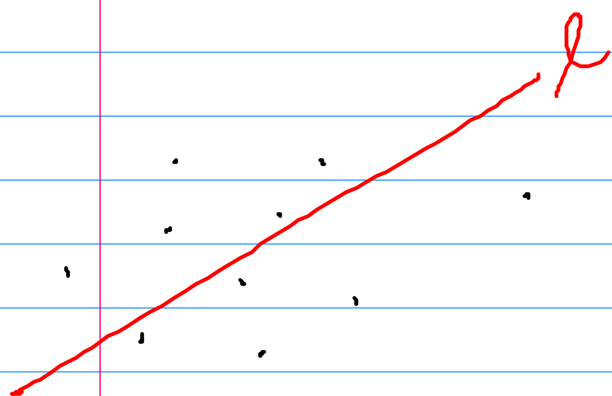
Primal plane



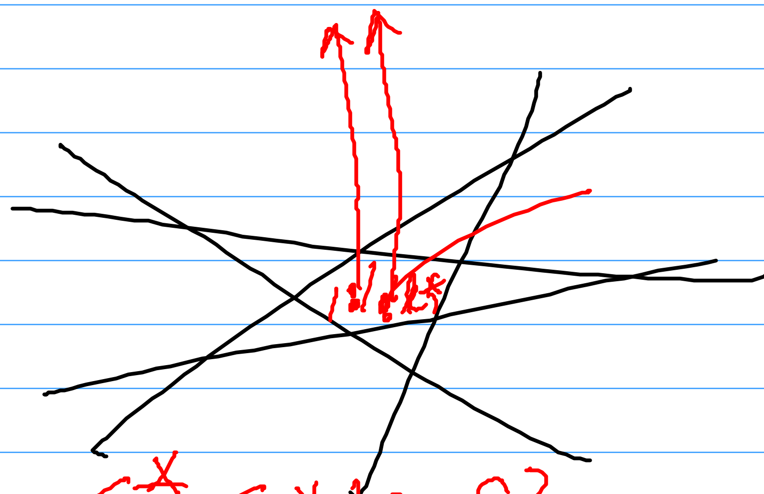
Dual plane

$p = (a, b) \longrightarrow p^*: \; y = ax + b$

$l: \; y = \alpha x + \beta \longrightarrow l^* = (-\alpha, \beta)$



Primal



$S^* = \{ p^* \mid p \in S \}$

Dual

Report all lines lying above l^*

Size		Time
$O(n^2)$	• Compute $A(S^*)$	$O(n^2)$
$O(n^2)$	• For each face $f \in A(S^*)$, w_f: # lines lying above f.	$O(n^2)$
$O(n^2)$	• Preprocess $A(S^*)$ for point-location queries	$O(n^2 \log n)$

Query procedure

$O(\log n)$

• Given ℓ, locate the face $f(\ell)$ containing ℓ^*

• Report $w_{f(\ell)}$