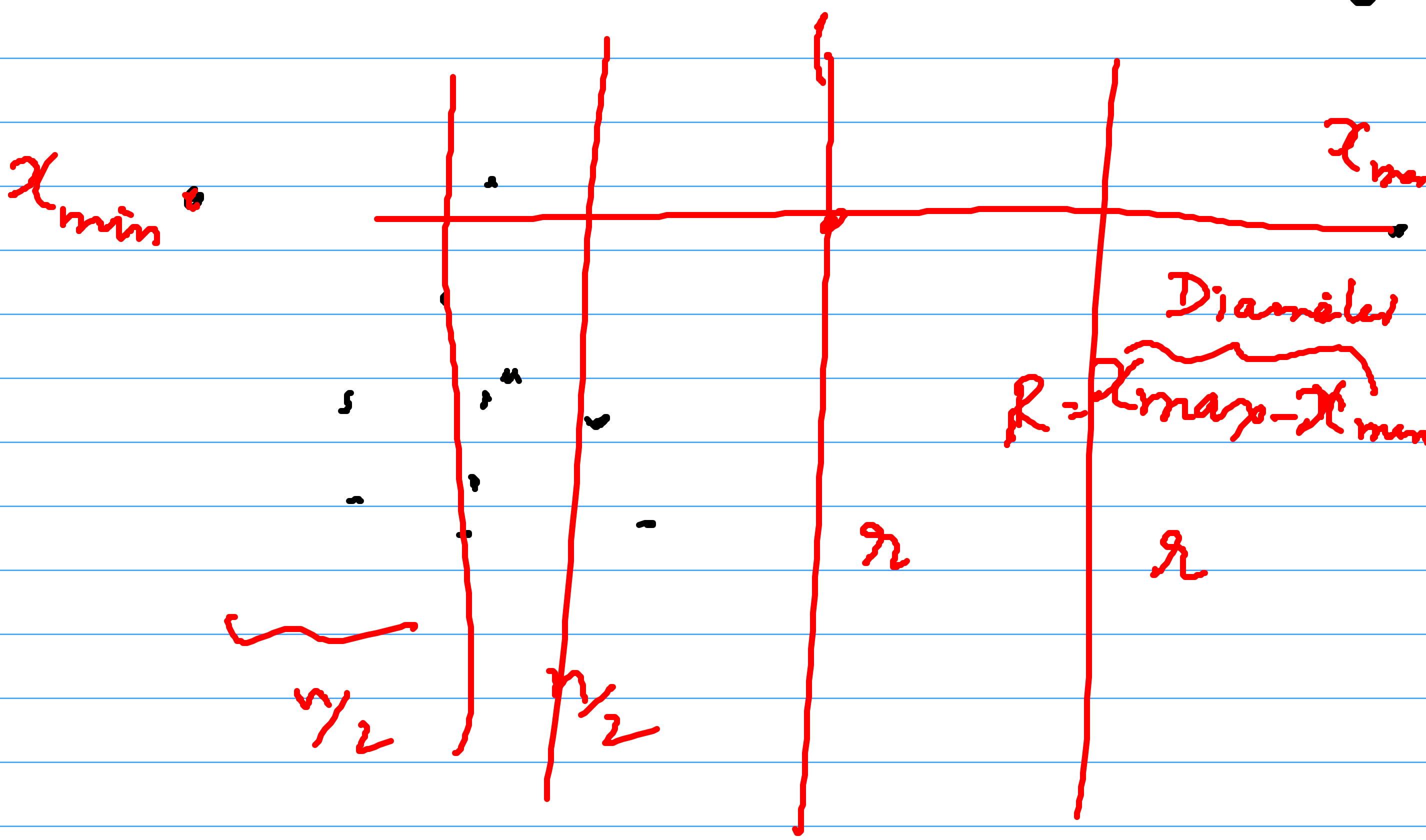
Computational Geometry : CSL 852

Lecture 29

Topic : Well-Separated Partitioning



$$\phi = \frac{R}{r} \quad : \text{spread}$$

$$\log \phi \gg \log n$$

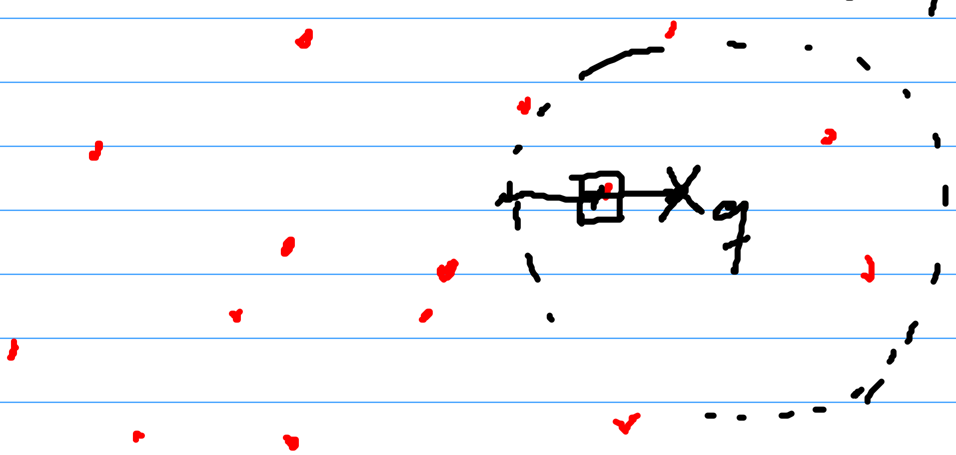
Build a data structure for nearest point query

ε. Approximate nearest neighbours

Given a set S of n points an ϵ-approximate neighbour of some arbitrary query point q is a point $p \in S$, s.t.

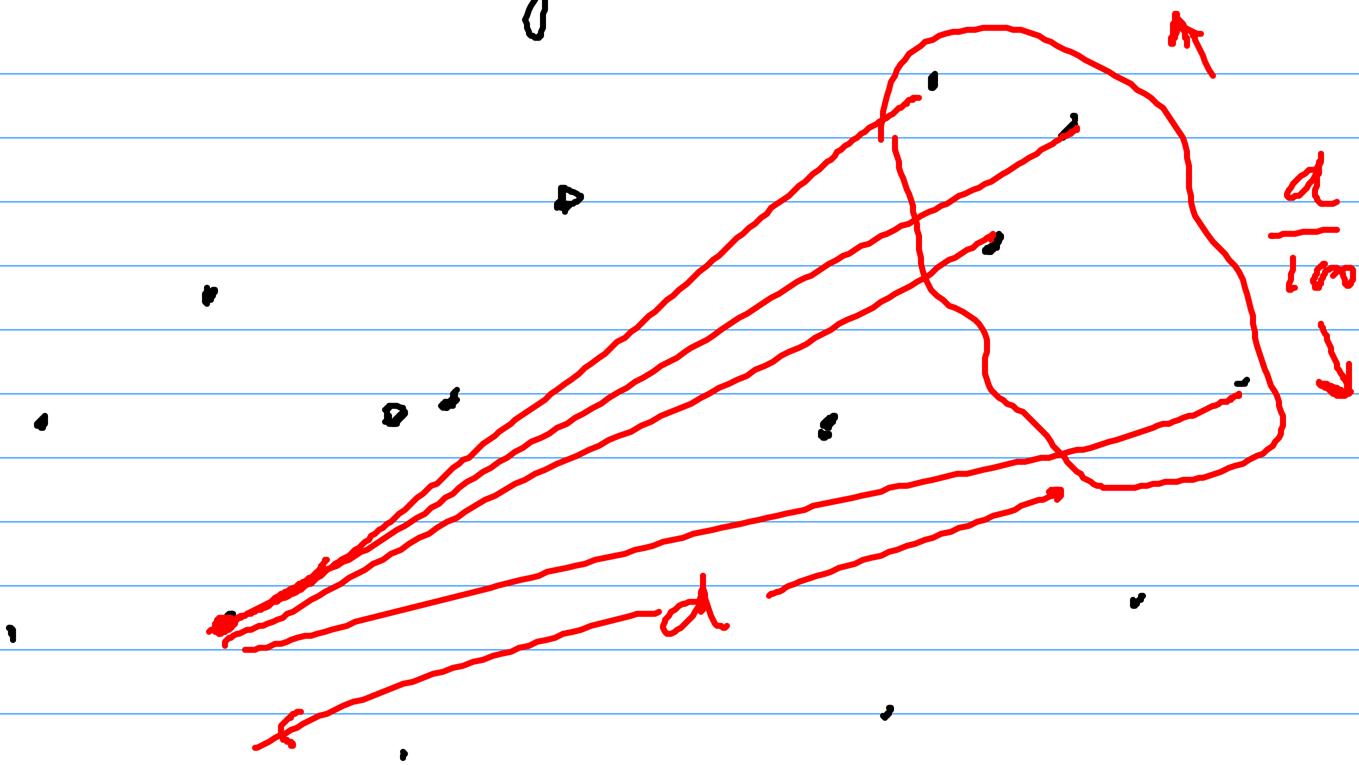
$$\mathrm{dist}(q, p) \leq (1+\varepsilon)\underbrace{\mathrm{dist}(q, S)}_{\min_{t \in S} \mathrm{dist}(q,t)}$$

$$\mathrm{dist}(P, Q) = \min_{p \in P,\, q \in Q} \mathrm{dist}(p, q)$$

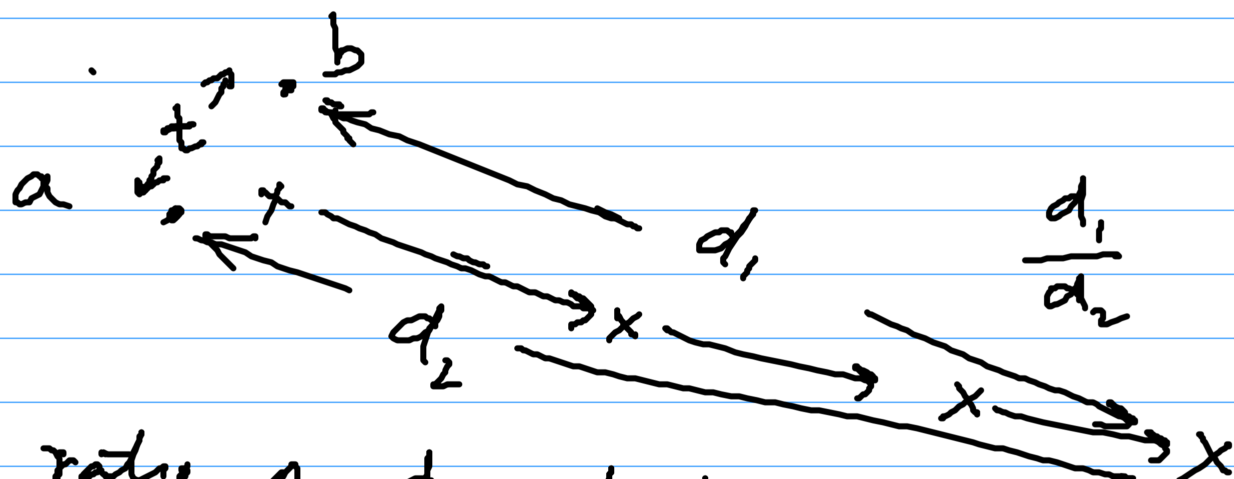


$\varepsilon = 1$

N-body simulation



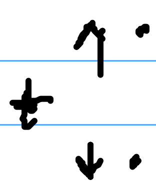
Compute all pairwise forces $\binom{N}{2}$



$\frac{d_1}{d_2}$ $\frac{d_2}{d_1}$

ratio of d_1 and d_2 decreases as x moves further

By Δ inequality



$d_1 \le d_2 + t$

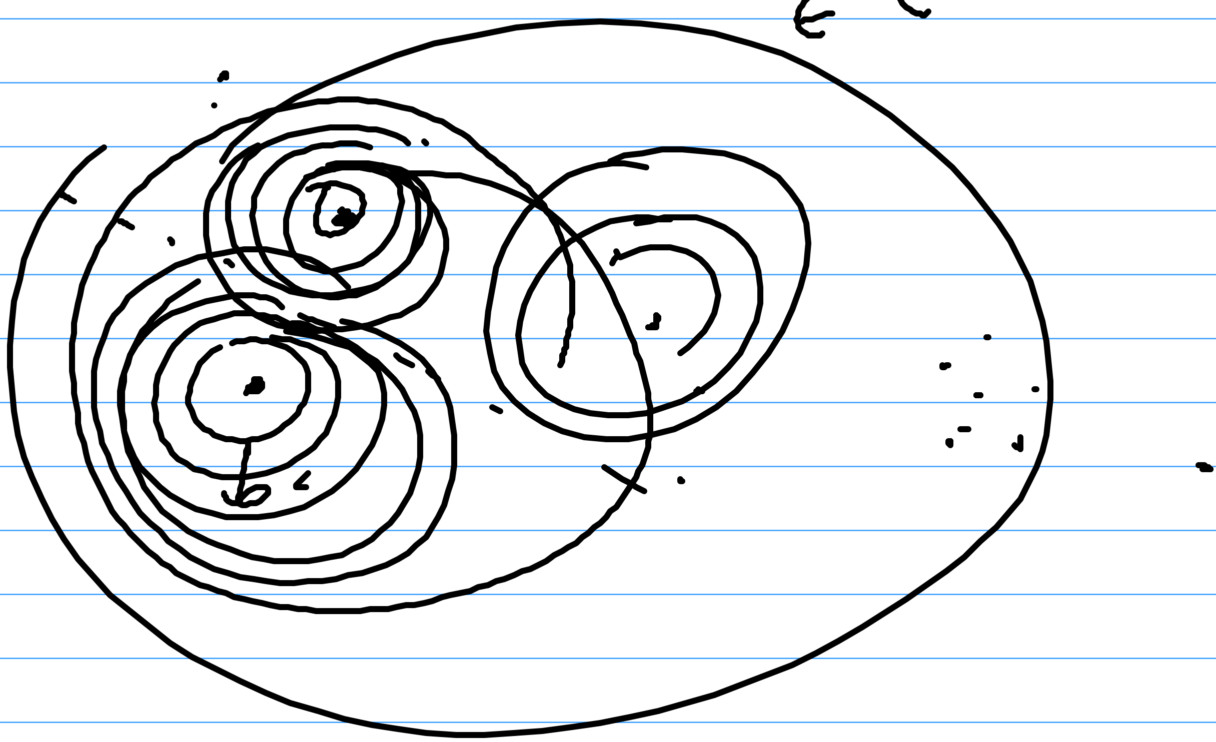
If we are interested in ϵ-approxi nearest neighbour, how far do we go before, a and b become indistinguishable.

$$d_2 \leq d_1 + t \quad , \quad d_1 \geq \frac{t}{\epsilon}$$

$$\min\{d_1, d_2\} \geq \frac{t}{\epsilon}$$

$$\max\{d_1, d_2\} \leq (1+\epsilon) \min\{d_1, d_2\} ?$$

$$d_2 \leq \frac{t}{\epsilon} + t = \frac{t}{\epsilon}(1+\epsilon)$$



t- spanner

Given a set P of n points in $\mathbb{E}^d$, we define a wt. graph $G = (P, E, w)$, such that for any $p, q \in P$

$$\|p - q\| \leq d_G(p, q) \leq t\,\|p-q\|$$

$t > 1$ stretch

$t = 1+\epsilon$

Intuitively : $|E| = f(\frac{1}{\epsilon})$ mon. increas

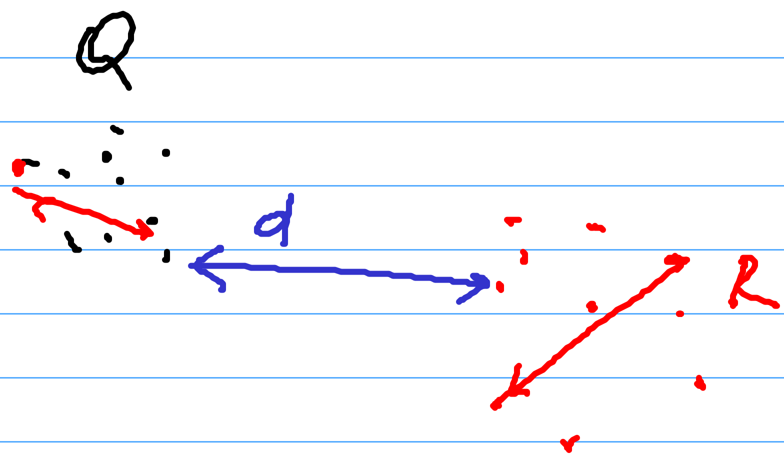
$$\left(\frac{1}{\epsilon}\right)^d \cdot n$$

Delaunay Triangulatn in 2D is a spanner wit $t < 3$

Claim : The MST of the graph G (corresponding to t spanner) is a t- MST of point set P.

A pair of point sets Q and R are $\frac{1}{\epsilon}$-separated if

$$\max\{\operatorname{diam}(Q), \operatorname{diam}(R)\} < \epsilon \cdot d(Q,R)$$



Pair Decomposition: For a point set P, a pair decomposition of P is a set of pairs

$$W = \{(A_1, B_1), (A_2, B_2) \ldots (A_s, B_s)\}$$

such that (i) $A_i, B_i \subset P$

(ii) $A_i \cap B_i = \emptyset$

(iii) $\bigcup_i A_i \otimes B_i = P \otimes P$

$$A \otimes B = \bigcup_{\substack{x \in A \\ y \in B}} (x, y)$$

ϵ- Well Separated Pair Decomposition

ϵ - WSPD