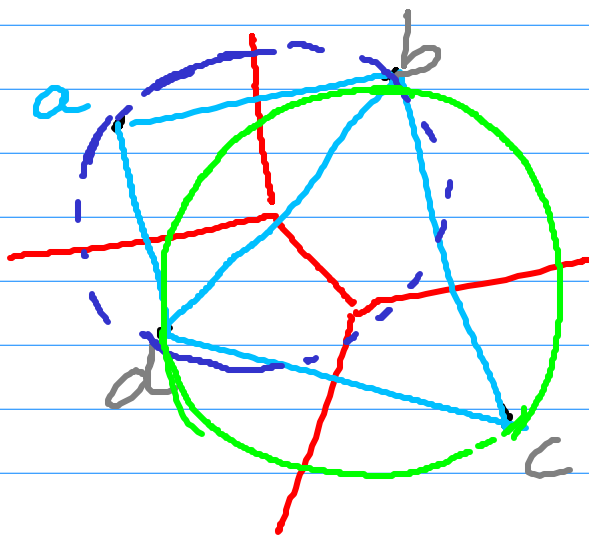


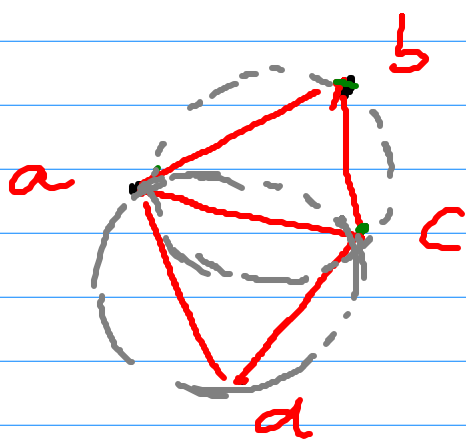
Lecture 19 : Delaunay Triangulation



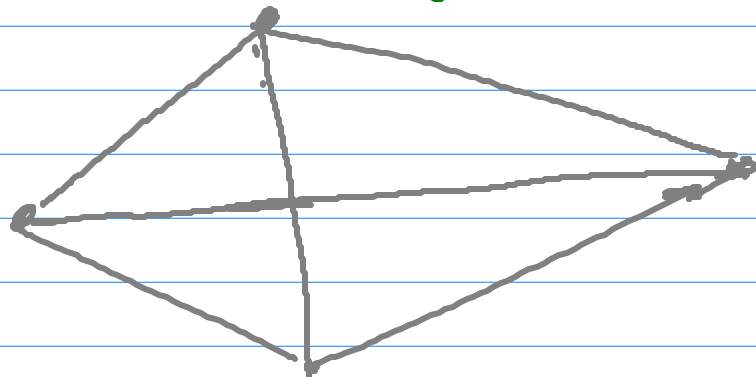
Delaunay
Triangulation.
The dual of
-the Voronoi Diagram

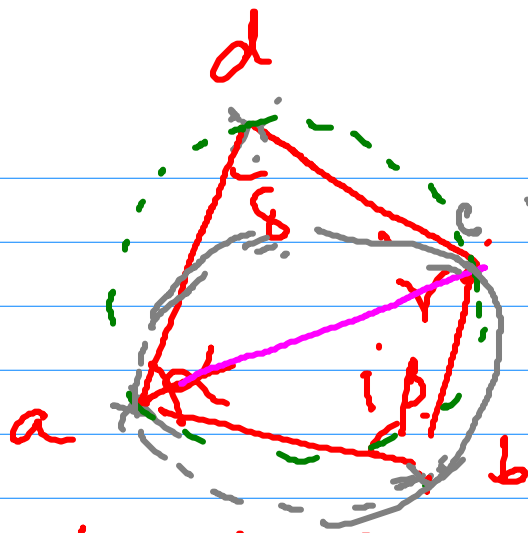
Delaunay disks : are defined as
-the interior of -the circumcircle
of a Delaunay Δ .

Delaunay disks, say $D \cap S = \emptyset$
(without considering the points
on the boundary)



X Delaunay Δ in





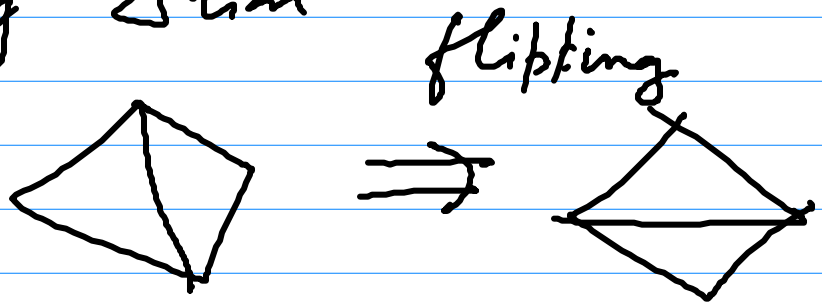
$$\alpha + \gamma \geq \pi$$

$$\delta + \beta < \pi$$

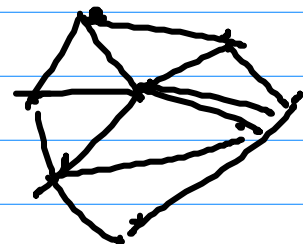
Flipping the diagonal

The point set S does not have 4 co-circular points (general position)

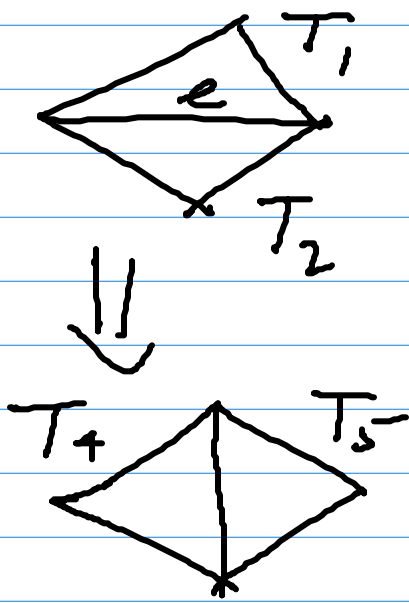
A simple algorithm for Delaunay Δ -tion



We start with some arbitrary Δ -tion. An arbitrary Δ -tion will contain $2n - 2 - k$ Δ^s where k is the # points on the convex hull of S .



We examine a pair of Δ^s that share an edge

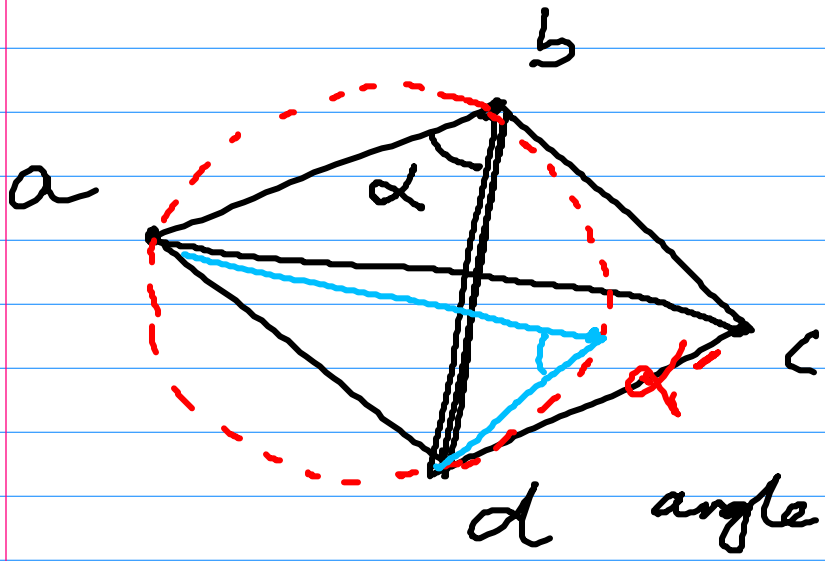


Does the $D(T_1)$ contain - the point of T_2 (on the opposite side of the common edge)

T_4 and T_5 are legal (diagonal is legal)

and now check if there exists another pair of Δ^s that are not legal.

until all diagonals are legal



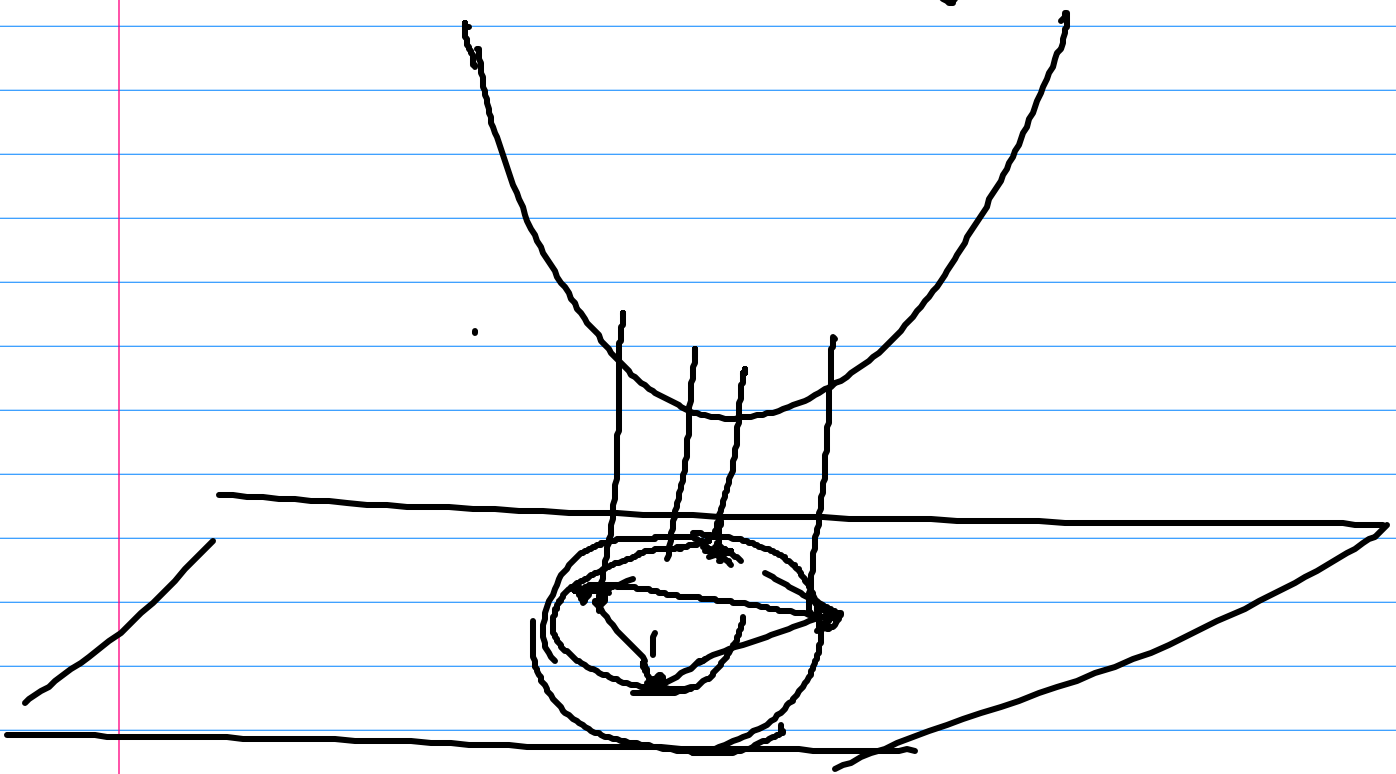
If C was on the circle

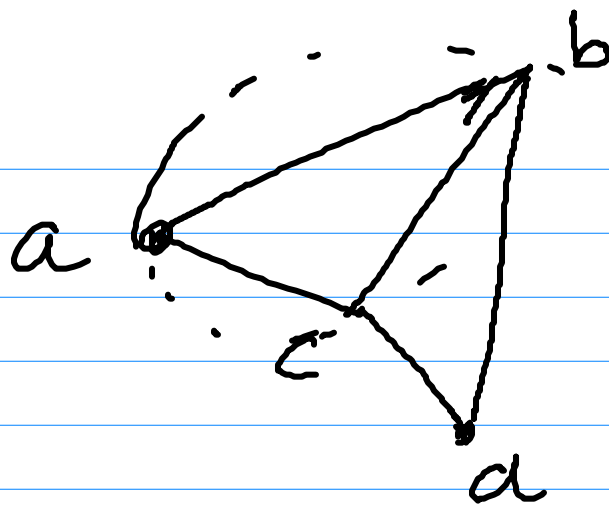
$\alpha' < \alpha$
 \Rightarrow the smallest of the Δ^s

The smallest angle of the triangulation defined by the "legal" edge is larger than the smallest angle of the Δ tion defined by the "illegal edge".

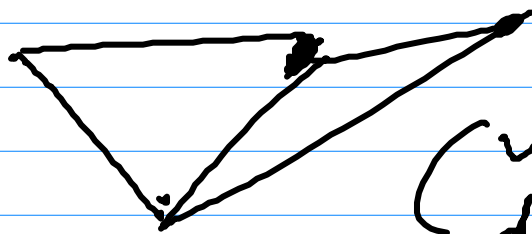
When we continue flipping the diagonals, the min angle of the triangulation increases.

Delannoy Δ tion maximises the min angle among all Δ tion.





The convergence
takes
 $O(n^2)$ edge
flips



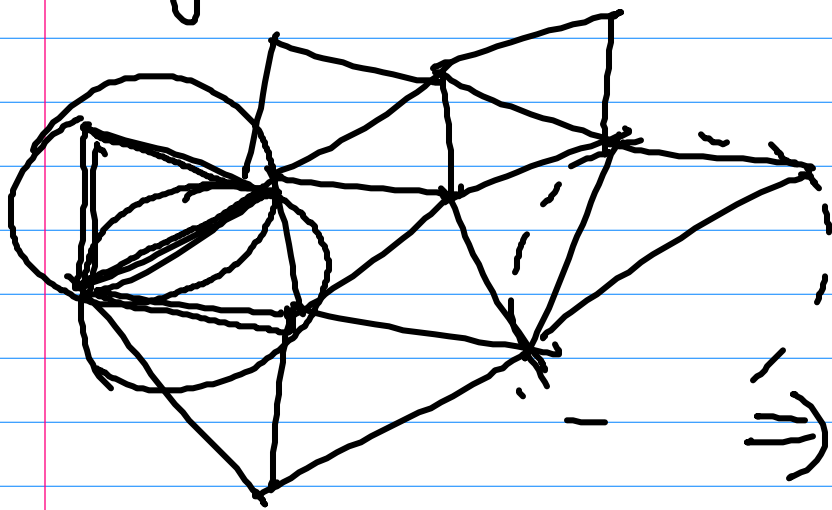
(using appropriate
data structures,
you can obtain $O(n^2)$ running

If you are given an
arbitrary Δ then, how quickly
can you verify that it is
a Delaunay Δ then

$O(n)$ Δ^s check if any
point lies within
the disk.

$\Theta(n^2)$ procedure.

If we can verify for every edge that the two Δ^s sharing the edge are legal, then it is a D.T.



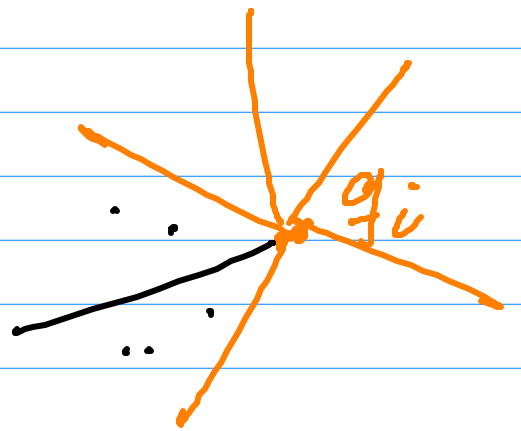
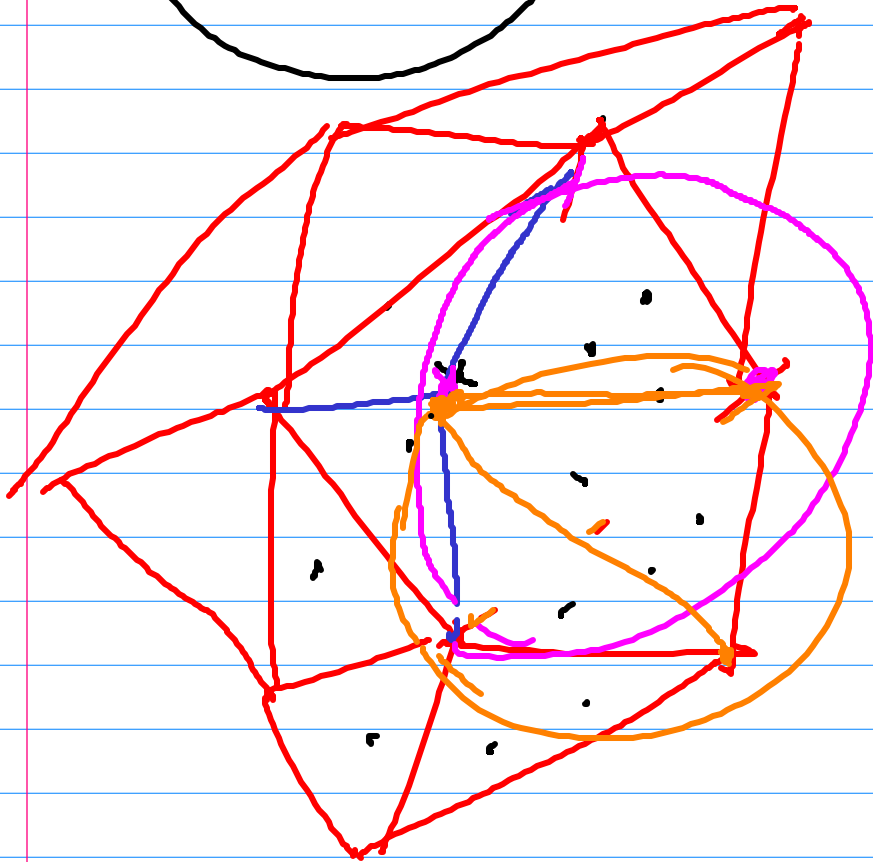
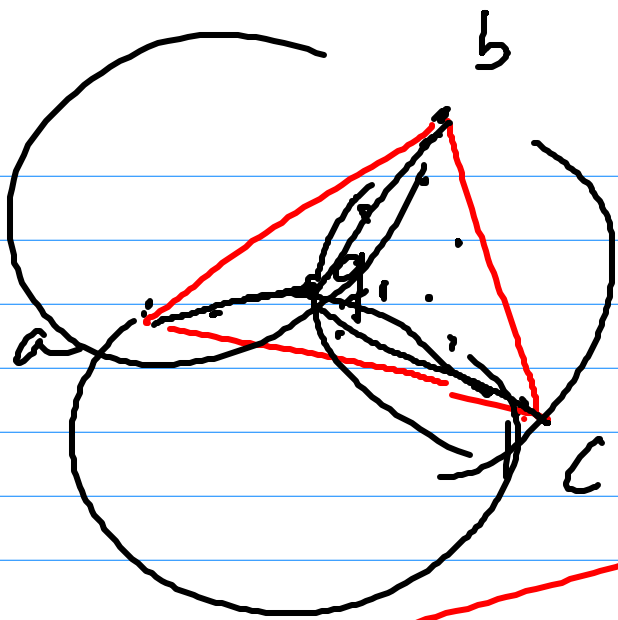
Local
 Delaunay
 property
 \Rightarrow Global property

Randomized incremental
Construction

Given the set of points $S = \{p_1, p_2, \dots, p_n\}$

first generate a random permutation of S , say $Q = \{q_1, q_2, \dots, q_n\}$

Enclose the points in some large Δ and then



→ Expected degree of g_i is $O(1)$

→ Main bearing the Δ to
 • Δ current D.T.

points not added

Claim: The randomized incremental construction for Delaunay Δ_{in} takes expected $O(n \log n)$ -time for any arbitrary set of n points.