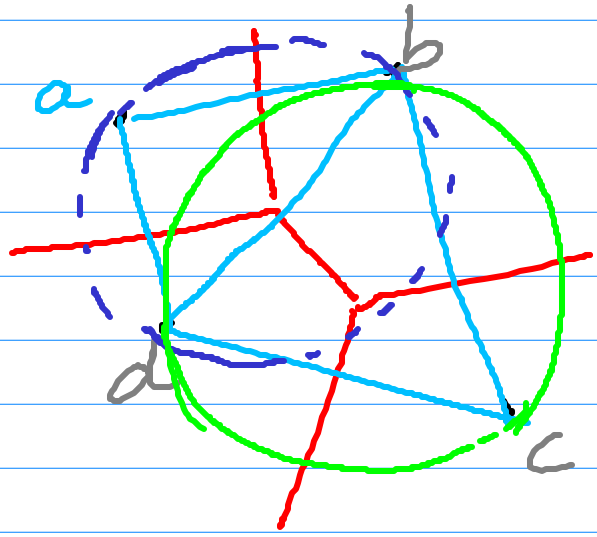
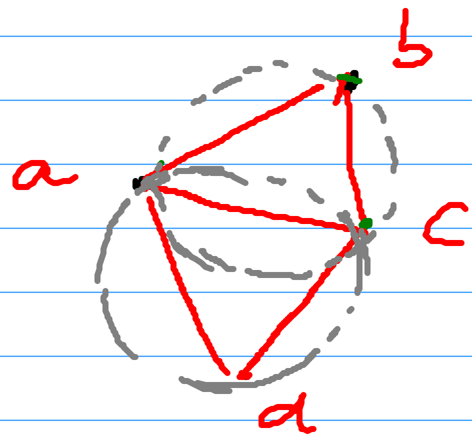
Lecture 19 : Delaunay Triangulation



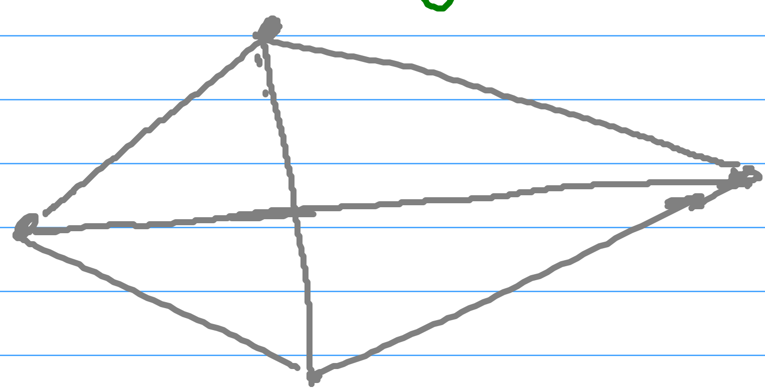
Delaunay Triangulation: The dual of the Voronoi Diagram

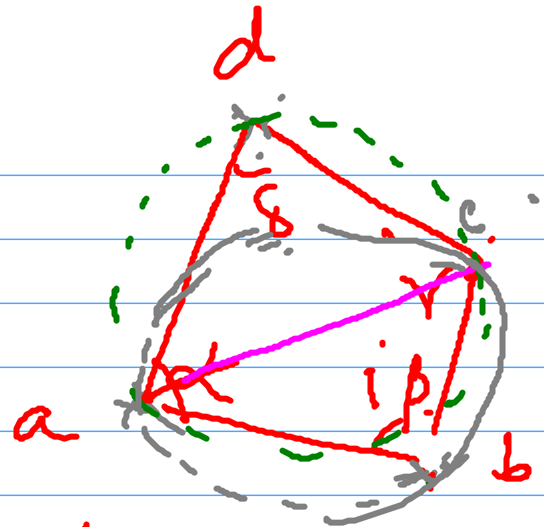
Delaunay disks: are defined as the interior of the circumcircle of a Delaunay Δ.

Delaunay disks, say $D \cap S = \emptyset$ (without considering the points on the boundary)



X Delaunay Δ-lation



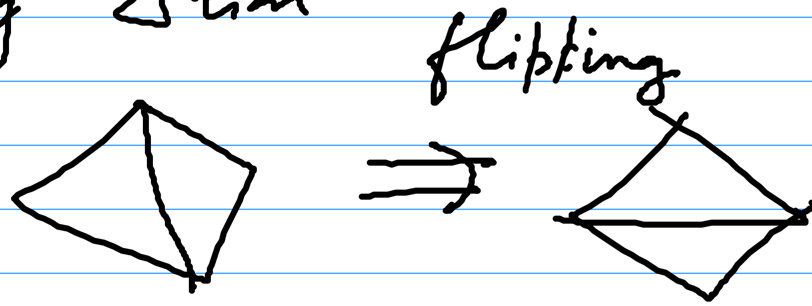


$\alpha + \gamma > \pi$
$\delta + \beta < \pi$

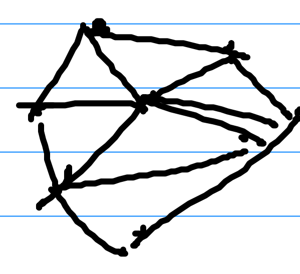
Flipping the diag

The point set S does not have 4 co-circular points (general position)

A simple algorithm for Delaunay Δtion

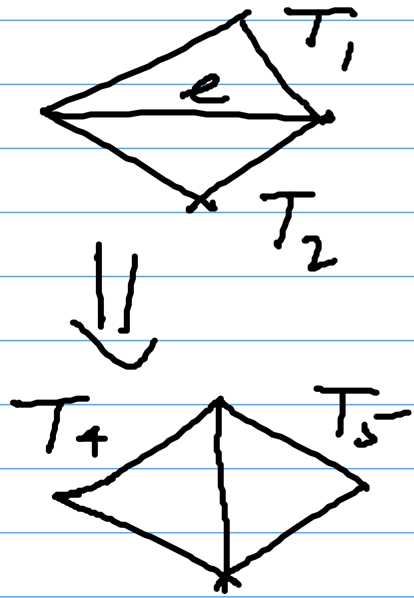


We start with some arbitrary Δtion. An arbitrary Δtion will contain $2n-2-k$ Δ^s where k is the # points on the convex hull of S.



We examine a pair of

Δ^s that share an edge

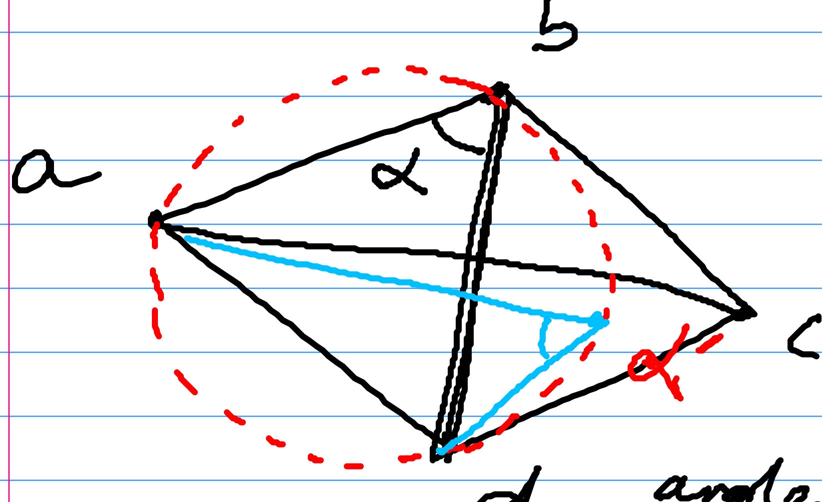


Does the $D(T_1)$ contain the point of T_2 (on the opposite side of the common edge)

T_4 and T_5 are legal (diagonal is legal)

and now check if there exists another pair of Δ^s that are not legal.

<u>until</u> all diagonals are legal



If C was on the circle

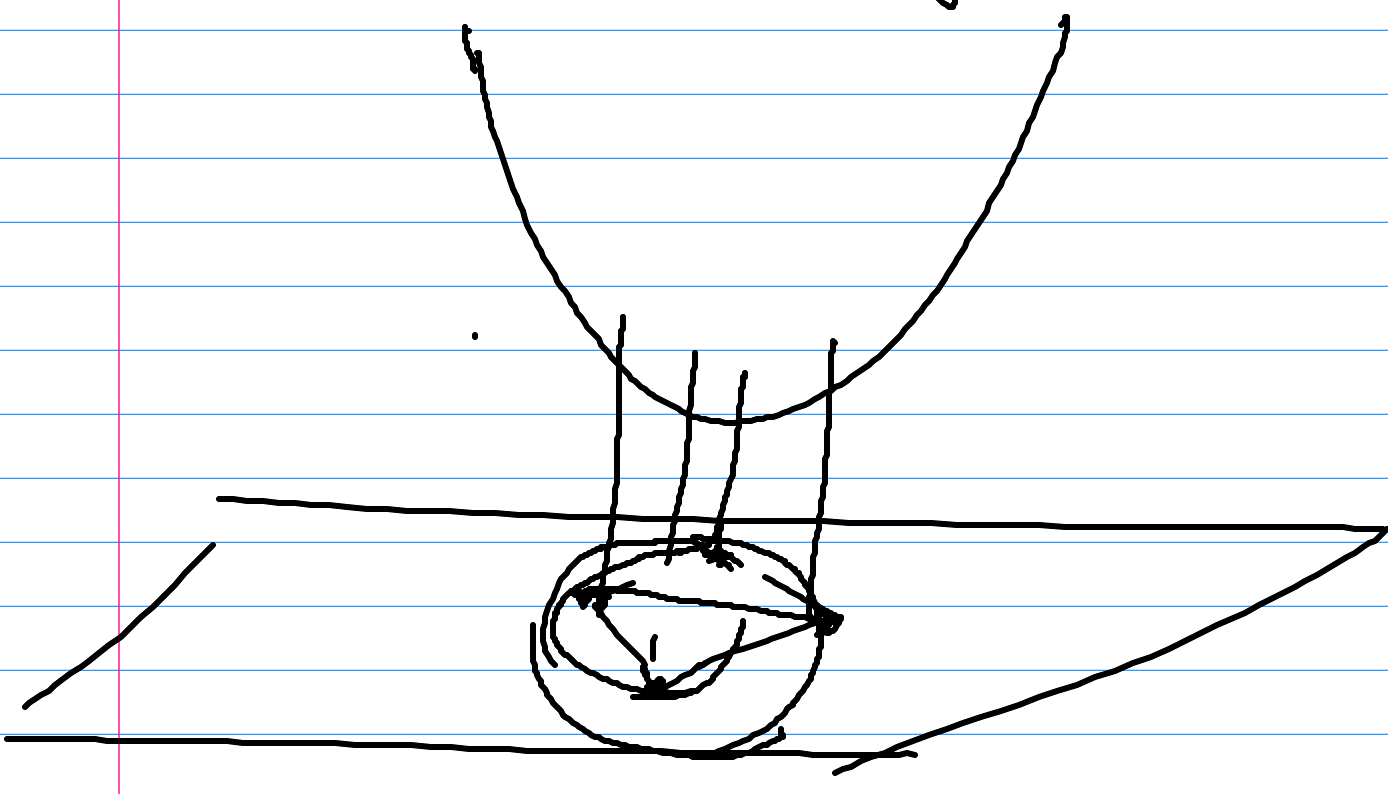
$\alpha' < \alpha$

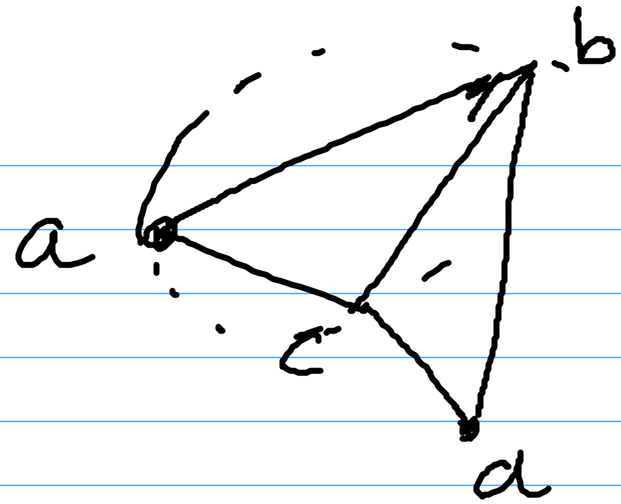
$\Rightarrow$ the smallest angle of the Δ^s

The smallest angle of the triangulation defined by the "legal" edge is larger than the smallest angle of the Δtion defined by the "illegal edge".

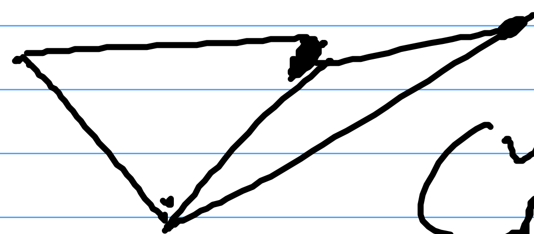
When we continue flipping the diagonals, the min angle of the triangulation increases.

Delaunay Δ-tion maximises the min angle among all Δ-tion.





The convergence takes $O(n^2)$ edge flips



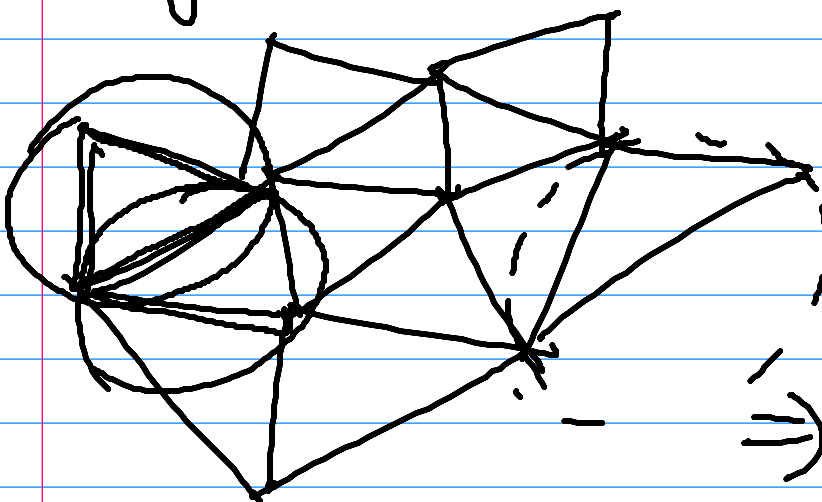
(using appropriate data structures, you can obtain $O(n^2)$ running

If you are given an arbitrary Δtion, how quickly can you verify that it is a Delaunay Δtion

$O(n)$ Δs — check if any point lies within the disk.

$\Theta(n^2)$ procedure.

If we can verify for every edge that the two Δ^s sharing the edge are legal, then it is a D.T.



Local Delaunay property

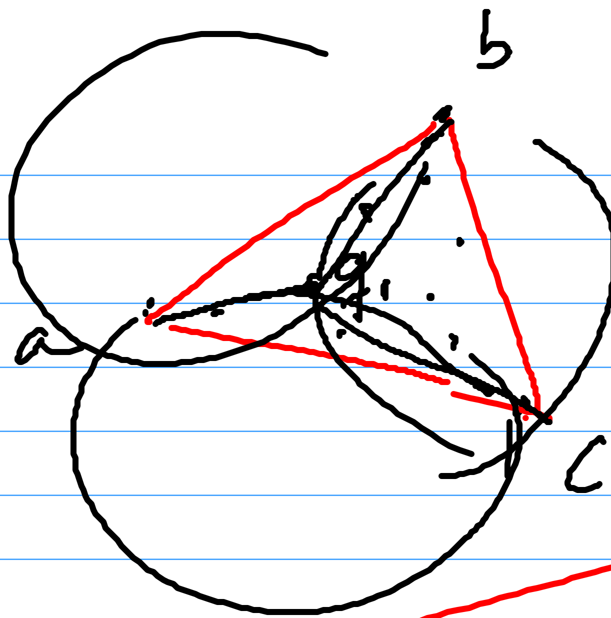
$\Rightarrow$ Global property

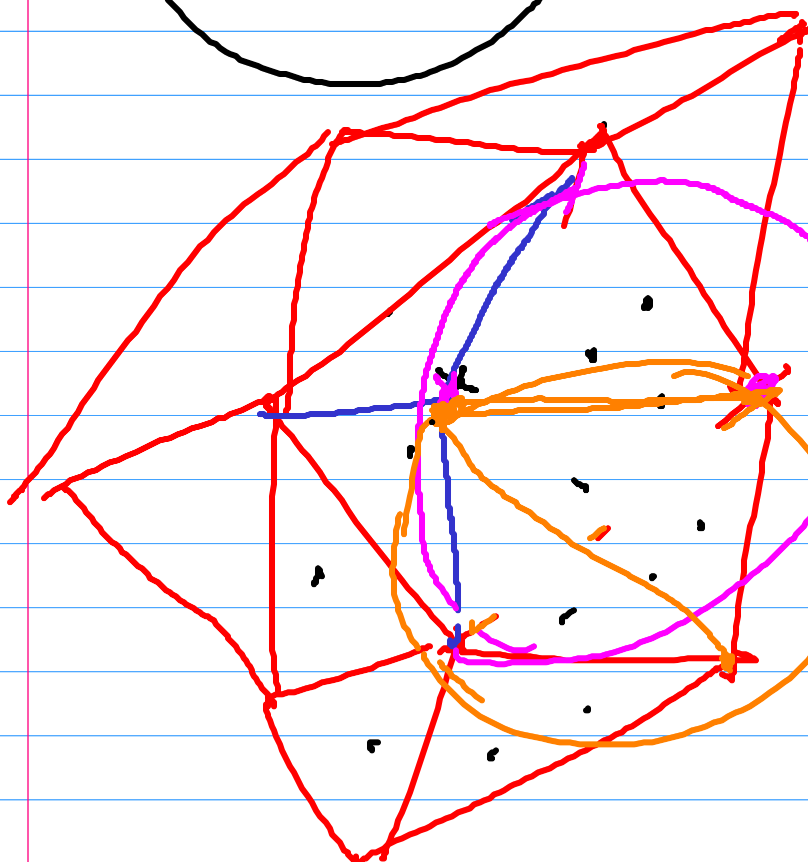
Randomized incremental construction

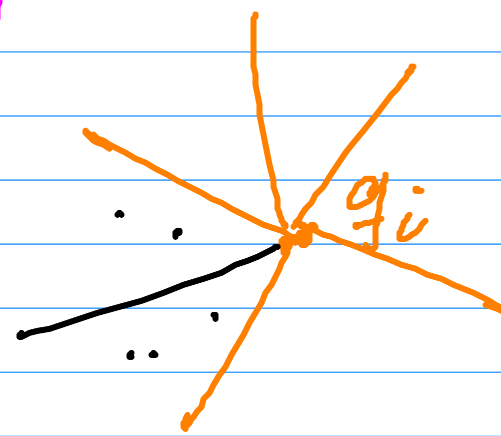
. Given the set of points $S = \{p_1, p_2, \cdots p_n\}$,

first generate a random permutation of S, say $Q = \{q_1, q_2, .. q_n\}$

Enclose the points in some large Δ and then







→ Expected degree of q_i is $O(1)$

→ Maintaining the Δ-tion

points not added · Δ current D.T.

<u>Claim</u> : The randomized incremental construction for Delaunay Δtion takes expected $O(n \log n)$ time for any arbitrary set of n points.