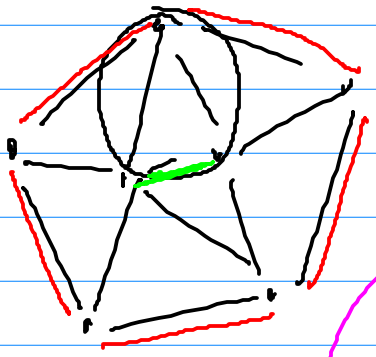
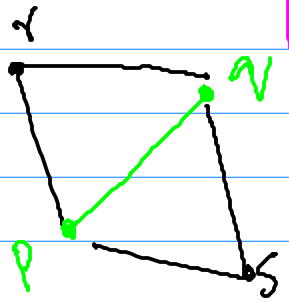
Lecture 19: Delaunay Triangulation



$S = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^2$

T: Triangulation of $conv(S)$

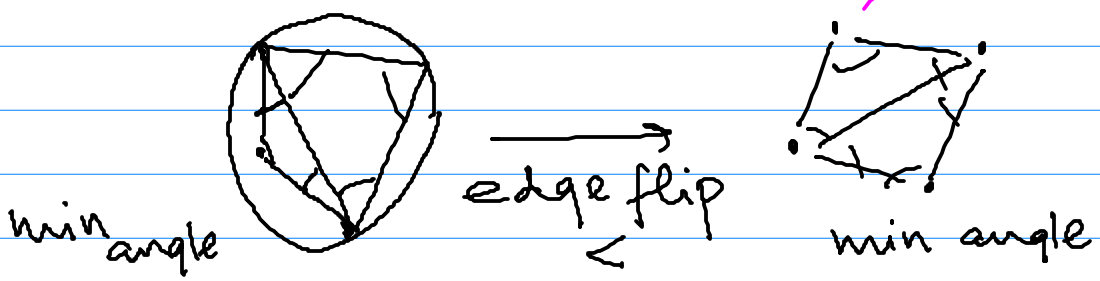
T is $DT(S) \iff \forall \triangle \in T$ ($\triangle pqr$) and $circ(pqr) \cap S = \phi$



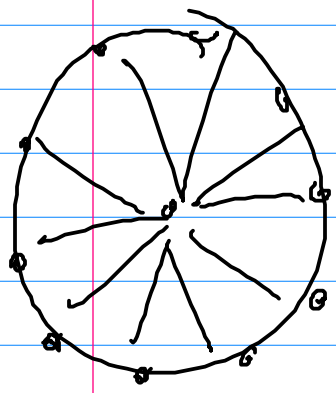
Definition: pq is **legal** if circle (p, q, r) does not contain s & $circ(p, q, s)$ does not contain r.

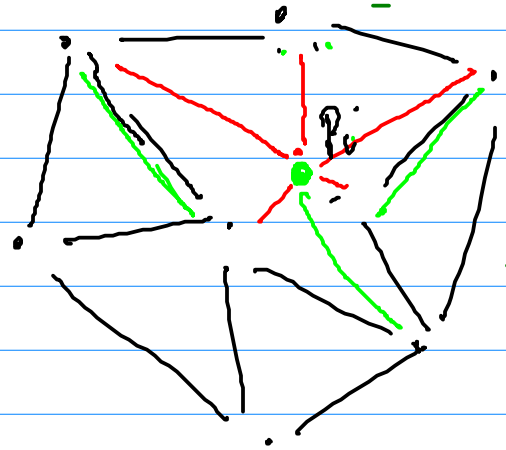
Lemma: If all edges in T are legal then T is $DT(S)$!

All edges in T are legal $\iff$ T is $DT(S)$

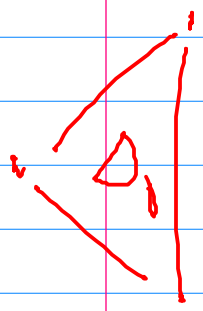


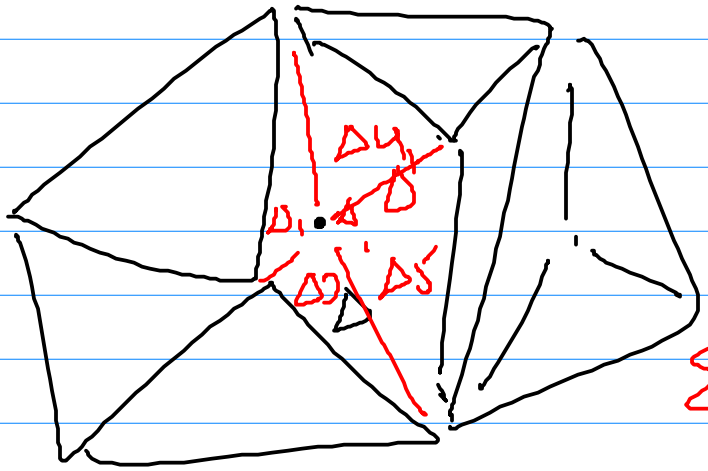
The flip sequence will terminate after $O(n^2)$ steps. (in 2D)

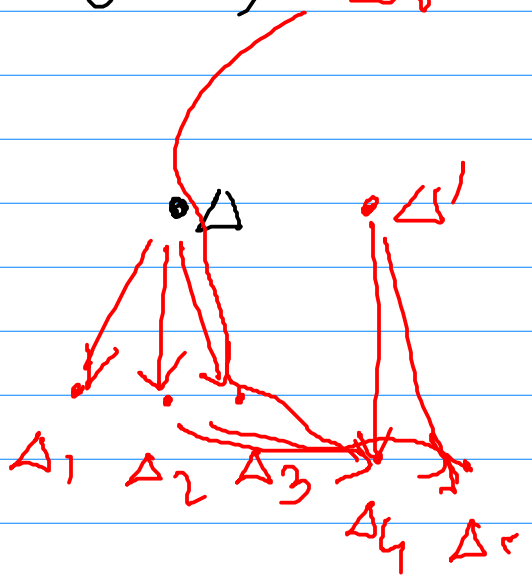




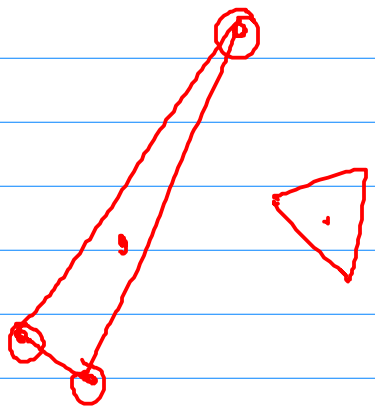
edge flips $\propto$ deg (p_i)

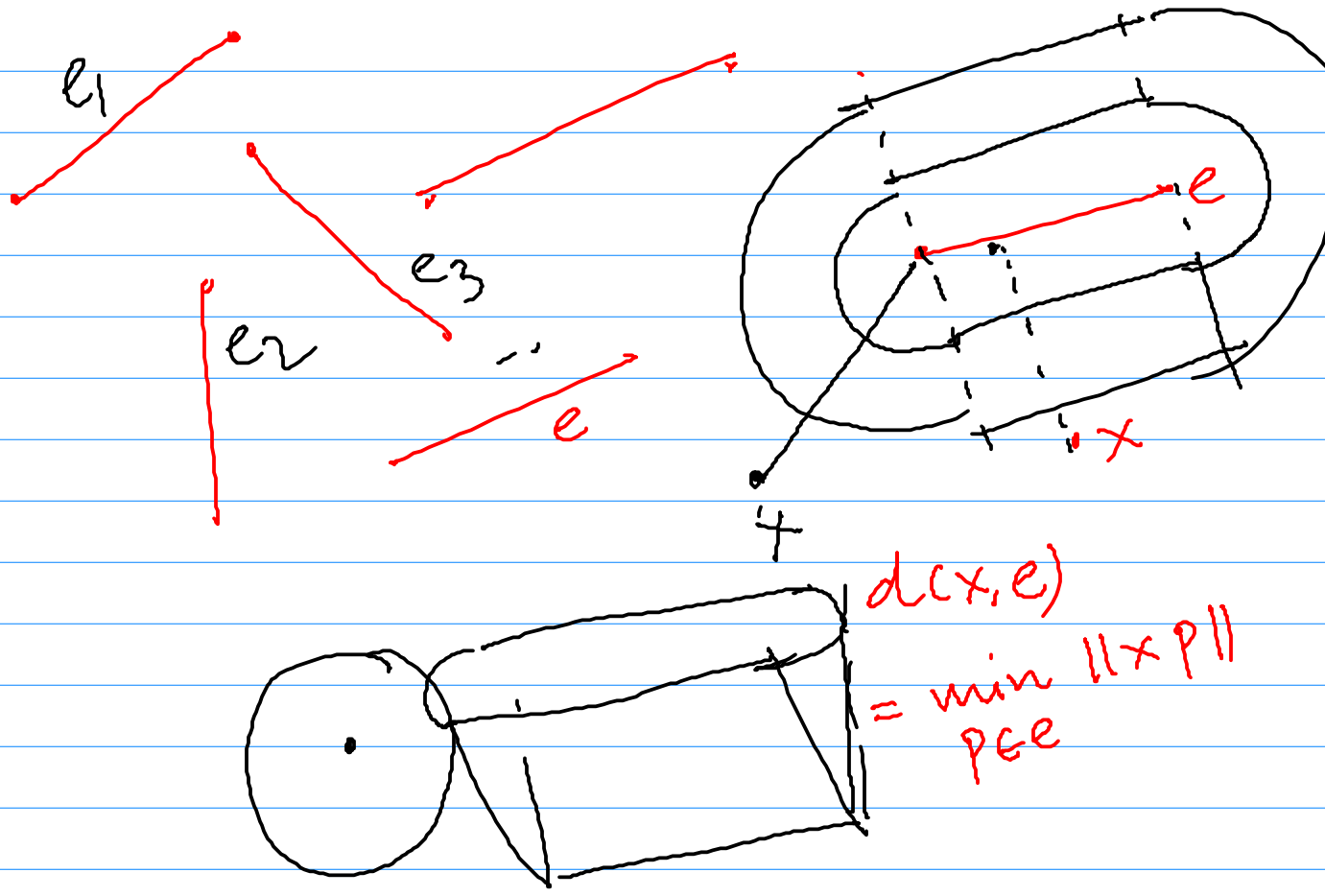






Expected depth $O(\log n)$



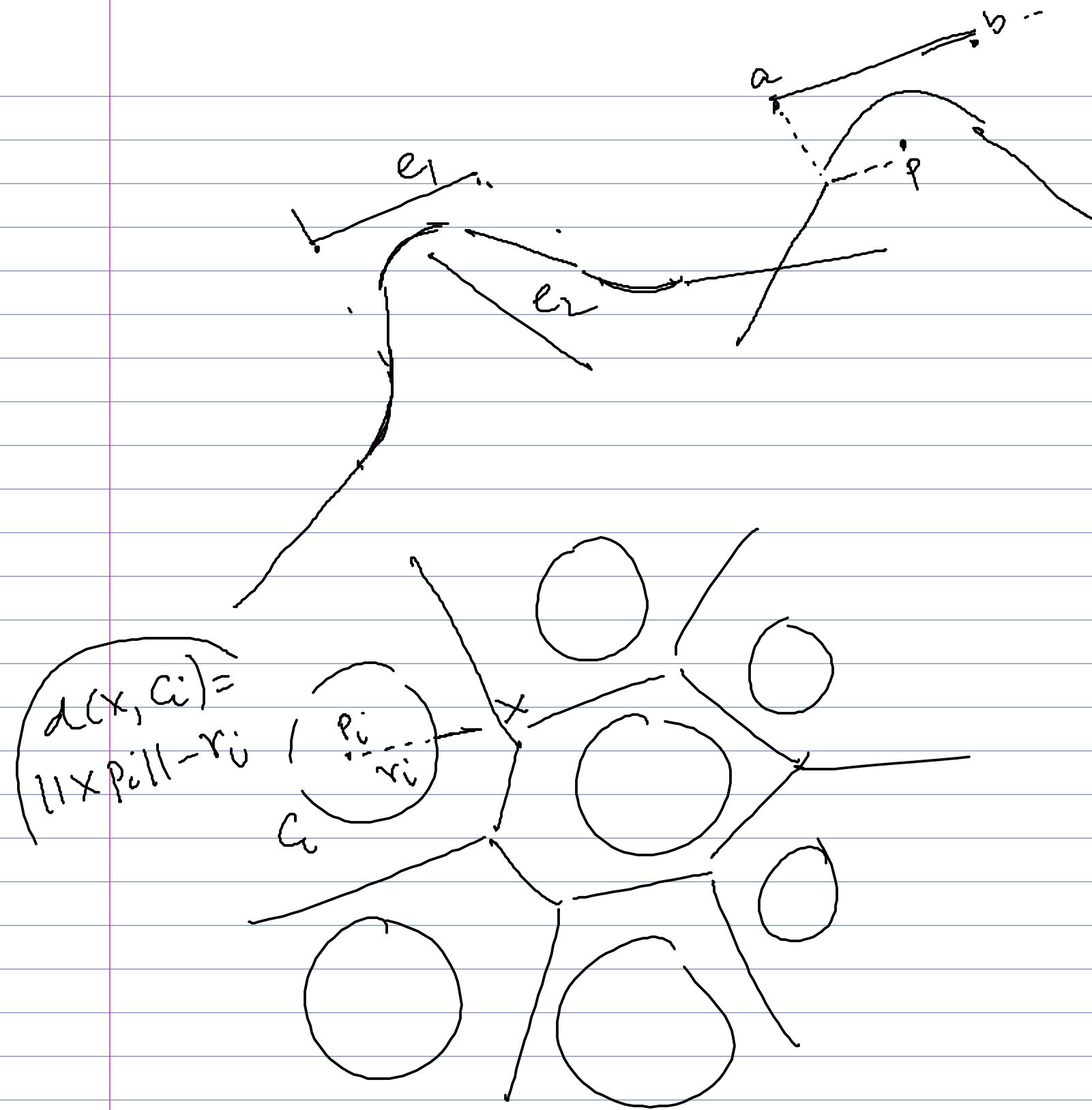


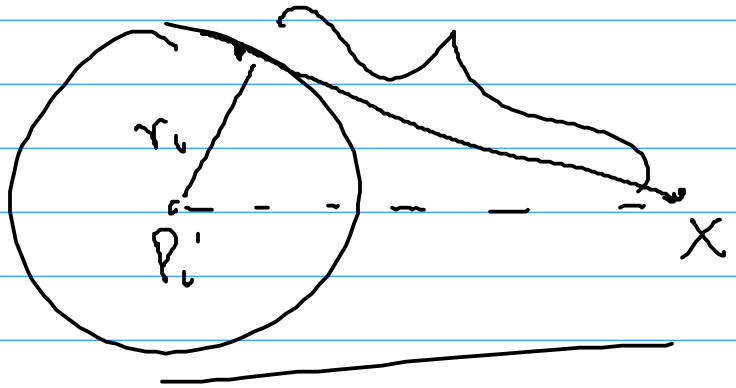
$$S = \{e_1, \ldots, e_n\}$$

$$f_i(x) = d(x, e_i)$$

$$F = \{f_1, \ldots, f_n\}$$

$Var(S)$: Minimization diagram of F.


b
a
p
e_1
e_2
$d(x, c_i) =$
$\|x - p_i\| - r_i$
p_i
r_i
x
c_i



$$d(x, c_i) = \sqrt{\|x - p_i\|^2 - r_i^2}$$

Power Diagram

$$\|x - p_i\|^2 - r_i^2$$