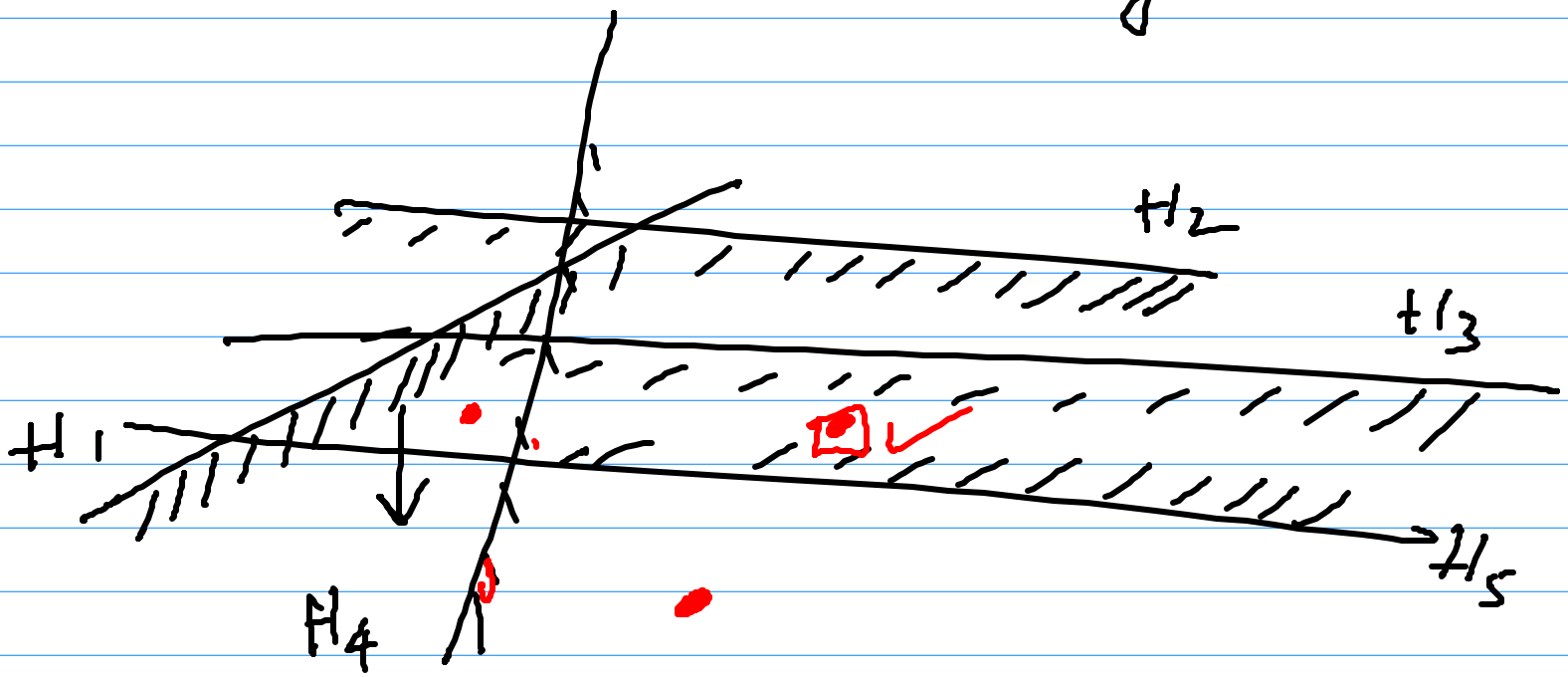
Computational Geometry Lecture 11

Topic: Intersection of half-planes and duality



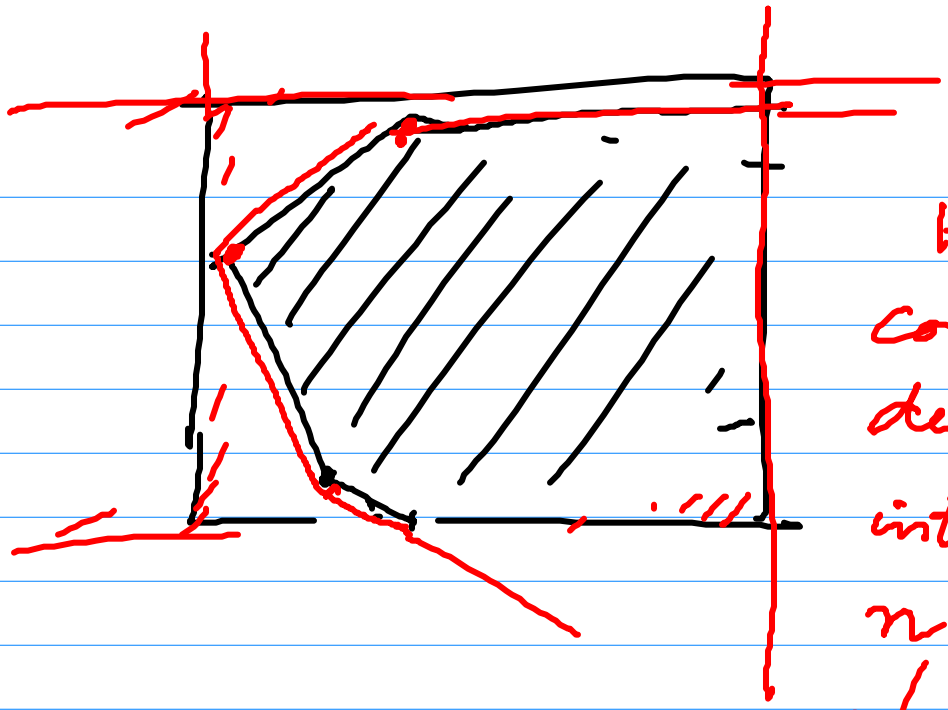
Given a set S of n half-planes compute the intersection $\bigcap_{i=1}^{n} H_i$

↙ region common to all half planes

Common region: convex polygon

caveats: (i) unbounded?

(ii) empty?

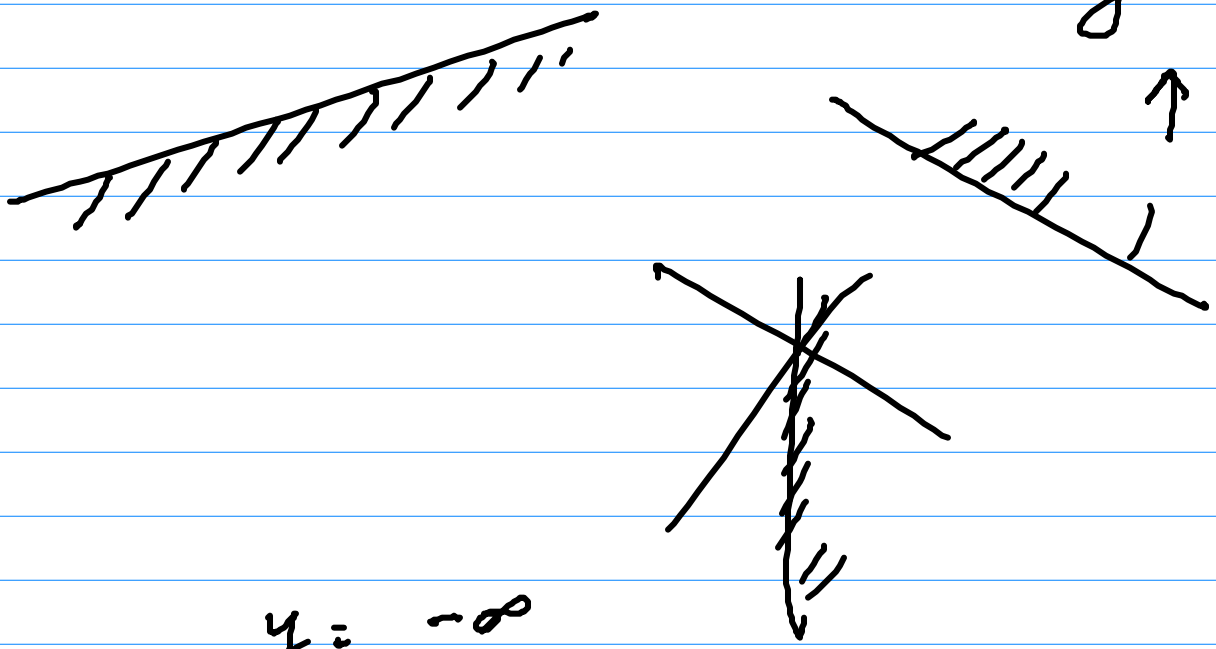


Higher dim convex polytope defined by intersection of n half-spaces

$\Theta\left(n^{\lfloor \frac{d}{2} \rfloor}\right)$

Distinguish between half-planes that are "downward" pointing vis a vis "upward pointing

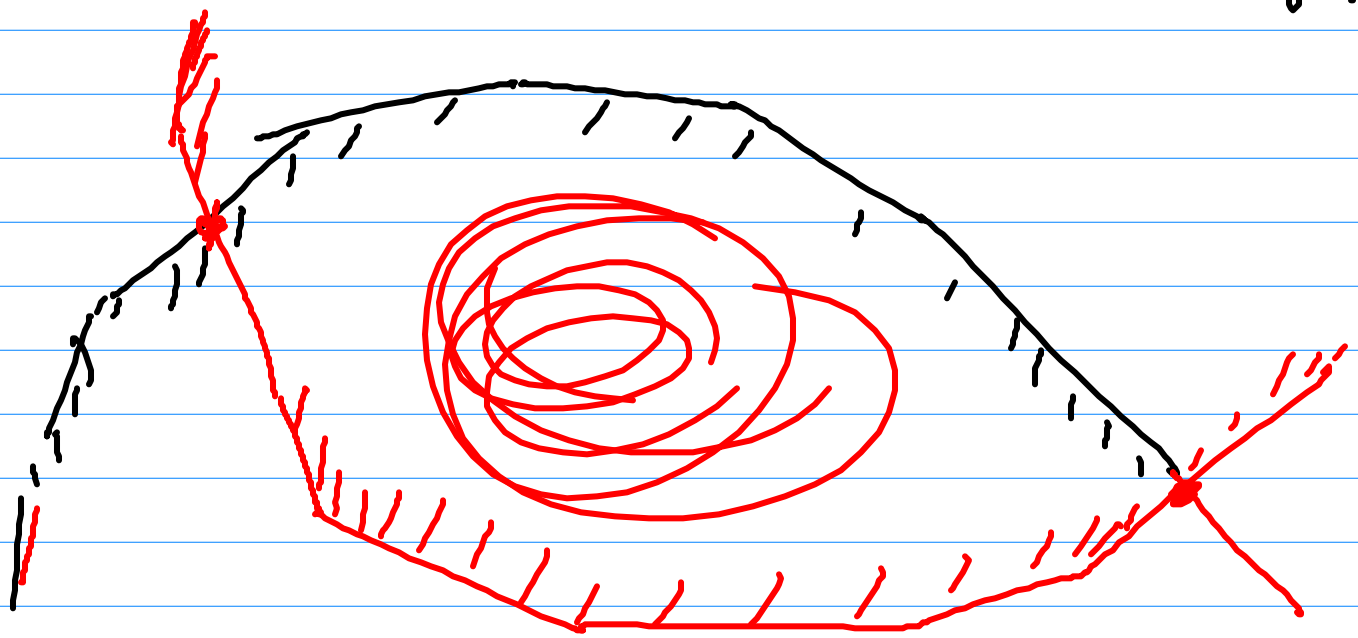
$y = +\infty$



$y = -\infty$

① Intersection of Downward half-planes

② " " Upward half-planes



$y \to -\infty$

Convex hulls : point input

Intersection : half-plane input

lines

Duality mapping D

D (points) → lines point : (a, b)

D (lines) → points line : $y = mx + c$

(m, c)

Desirable properties of D

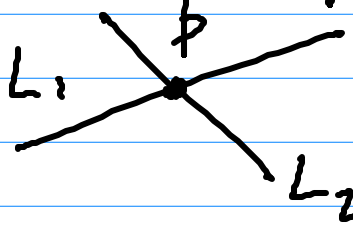
(1) $D(D(x)) = x$ x · point or line

(2) D is 1-1

(3) Incidence : Consider a point p and a line L

If p is incident on L, then $D(L)$ is incident on $D(p)$

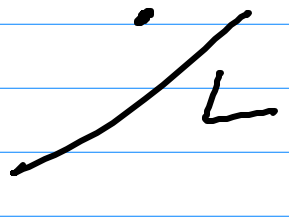
(4) If L_1 and L_2 intersect in p



then $D(p)$ should pass through $D(L_1)$ and $D(L_2)$

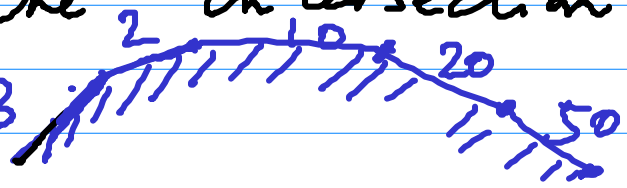
(5) Above-Below property

If p lies above L then $D(L)$ will below $D(p)$

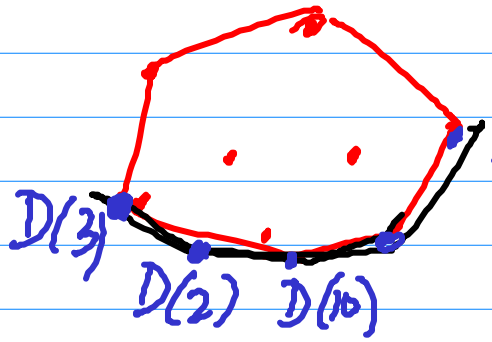


H

Given a set of downward half-planes, let I(H) denote the intersection (which is a convex chain)



Consider the duals of the lines describing the half-planes H, denote it by S. Let CH(S) denote the convex hull of S



Claim

The downward convex chain describing I(H) is in 1-1 correspondence with the points (duals of lines describing H) on the lower hull.